

Formulas

Line Integral formula

$$\int_C \vec{F} \cdot d\vec{r} = \int_a^b \vec{F}(\vec{r}(t)) \cdot \vec{r}'(t) dt$$

Flux Across a Cylinder

Let S be the surface of a cylinder of radius R centered on the z -axis, with base at $z = a$ and top at $z = b$. Then

$$\int_S \vec{F} \cdot d\vec{A} = \int_0^{2\pi} \int_a^b \vec{F}(R, \theta, z) \cdot (\cos \theta \vec{i} + \sin \theta \vec{j}) R dz d\theta$$

Flux Across a Sphere

Let S be the surface of a sphere of radius R centered at the origin. Then

$$\int_S \vec{F} \cdot d\vec{A} = \int_0^{2\pi} \int_0^\pi \vec{F}(R, \theta, \phi) \cdot (\sin \phi \cos \theta \vec{i} + \sin \phi \sin \theta \vec{j} + \cos \phi \vec{k}) R^2 \sin \phi d\phi d\theta$$

Flux Across a Surface defined by $z = f(x, y)$

Let S be the surface in three space defined by $z = f(x, y)$ with x and y in the region R in the xy -plane. Assume S is oriented upward. Then

$$\int_S \vec{F} \cdot d\vec{A} = \int_R \vec{F}(x, y, f(x, y)) \cdot (-f_x \vec{i} - f_y \vec{j} + \vec{k}) dx dy$$

Flux Across a Parametrized Surface

Let S be the surface in three space parametrized by $\vec{r}(s, t)$ where $a \leq s \leq b$ and $c \leq t \leq d$. Then

$$\int_S \vec{F} \cdot d\vec{A} = \int_c^d \int_a^b \vec{F}(\vec{r}(s, t)) \cdot \left(\frac{\partial \vec{r}}{\partial s} \times \frac{\partial \vec{r}}{\partial t} \right) ds dt$$

We choose the parametrization so that $\frac{\partial \vec{r}}{\partial s} \times \frac{\partial \vec{r}}{\partial t}$ is never 0 and points in the direction of the orientation of S .

Curl of a 3d Vector Field

Let $\vec{F} = F_1 \vec{i} + F_2 \vec{j} + F_3 \vec{k}$. Then

$$\text{curl } \vec{F} = \left(\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z} \right) \vec{i} + \left(\frac{\partial F_1}{\partial z} - \frac{\partial F_3}{\partial x} \right) \vec{j} + \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) \vec{k}.$$

Four Big Theorems

Theorem 0.1. [Fundamental Theorem of Line Integrals] *Let C be a curve with endpoints P and Q . Let \vec{F} be a vector field with $\vec{F} = \text{grad}f$. Then*

$$\int_C \vec{F} \cdot d\vec{r} = f(Q) - f(P).$$

Theorem 0.2. [Green] *Let C be a simple closed curve in the xy -plane, oriented counterclockwise, and let R be the region of the xy -plane enclosed by C . Let $\vec{F} = F_1\vec{i} + F_2\vec{j}$ be a differentiable vector field on R and on C . Then*

$$\int_C \vec{F} \cdot d\vec{r} = \int_R \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) dx dy.$$

Theorem 0.3. [The Divergence Theorem] *Let S be a closed surface, oriented outward, and let W be the solid enclosed by S . Let \vec{F} be a differentiable vector field on W and S . Then*

$$\int_S \vec{F} \cdot d\vec{A} = \int_W \text{div}\vec{F} dV.$$

Theorem 0.4. [Stokes] *Let S be a surface with boundary C . Assume C is oriented so that the surface S is on the left as you travel along C . Let \vec{F} be a differentiable vector field on S and C . Then*

$$\int_C \vec{F} \cdot d\vec{r} = \int_S \text{curl}\vec{F} \cdot d\vec{A}.$$