

**Math 203, Exam #1**  
**Fall 2008**

**Name:** \_\_\_\_\_

Please write out the full pledge and sign below.

**1. True/False** Decide whether each of the following statements is true or false. **Please give a short explanation for your answer.**

**a. T F** In some cases, a matrix may be row reduced to two different matrices in reduced echelon form, using different sequences of row operations.

**b. T F** If one row of the echelon form of the augmented matrix for a system of equations is  $[0 \ 0 \ 0 \ 0 \ 0 \ 2]$ , then the system is inconsistent.

**c. T F** The span of two vectors  $\vec{v}$  and  $\vec{w}$  is a plane through the origin.

**2.** A system of equations has an augmented matrix of the form:

$$\begin{bmatrix} 1 & a & 0 & 0 & b \\ 0 & 0 & 1 & 0 & c \\ 0 & 0 & 0 & 1 & d \end{bmatrix}$$

Describe the solutions set.

**3.** Suppose you have a consistent system of 2 linear equations with 3 variables  $x_1$ ,  $x_2$  and  $x_3$ . Find all possible forms for the echelon form for the augmented matrix. For each form, determine which variables  $x_1$ ,  $x_2$  and  $x_3$  are free. [**Note: be sure to only include forms that represent consistent systems**]

4. Consider the system of linear equations:

$$\begin{array}{rccccrcr} x_1 & + & 2x_2 & + & x_3 & & = & 4 \\ -2x_1 & - & 4x_2 & + & x_3 & - & x_4 & = & -9 \\ 3x_1 & + & 6x_2 & + & 2x_3 & & & = & 1 \end{array}$$

a. Write this system of equations as a matrix equation.

b. Write this system of equations as a vector equation.

c. Find the augmented matrix of this system.

d. Use row operations to find the reduced echelon form of this matrix.

e. Use your answer to part **d** to find the solution set to the system of equations. Please give your answer in parametric form.

5. Let  $\vec{v} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ , and  $\vec{w} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$ .

a. Compute  $2\vec{v} - 3\vec{w}$

b. Which of the following vectors are in the span of  $\vec{v}$  and  $\vec{w}$ ?

(i)  $\begin{bmatrix} 2 \\ 4 \\ 0 \end{bmatrix}$

(ii)  $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

(iii)  $\begin{bmatrix} 1 \\ 8 \\ 2 \end{bmatrix}$

c. For which value(s) of  $h$  is the vector  $\begin{bmatrix} 1 \\ 2h \\ h \end{bmatrix}$  in the span of  $\vec{v}$  and  $\vec{w}$ ?

6. Determine which of the following matrices are in echelon form and which are in reduced echelon form, and which are in neither form.

a.  $\begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$

b.  $\begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$

c.  $\begin{bmatrix} 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$

d.  $\begin{bmatrix} 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$

e.  $\begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$

f.  $\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$