Math 203, Exam #1 Review **Fall 2009**

Your exam will cover sections 1.1-1.5, 1.7-1.9. In addition to these problems, you should look over your homework - including the true/false questions. Be prepared to justify your answers.

1. Consider the system of linear equations:

$$\begin{array}{rcrcrcl} 2x_1 & + & x_2 & - & 2x_3 & = & 0 \\ x_1 & - & x_2 & - & x_3 & = & 0 \\ -2x_1 & + & x_2 & + & 2x_3 & = & 0 \end{array}$$

a. Find the augmented matrix of this system.

b. Use row operations to find the reduced echelon form of this matrix.

c. Use your answer to part **b** to find the solution set to the system of equations.

2. Suppose you have a system of 3 linear equations with 4 unknowns.

a. Find all possible forms for the echelon form for the augmented matrix.

b. Which of these forms represent cases where there is no solution? Exactly one solution? 2 free variables?

3. A system of equations has an augmented matrix of the form:

$$\begin{bmatrix} \blacksquare & * & * & * \\ 0 & 0 & \blacksquare & * \end{bmatrix}$$

What can you say about the solution set, algebraically and geometrically?

4. State the definition of a consistent system of equations.

5. Let $\vec{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, and $\vec{w} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$.

a. Compute $\vec{v} - 2\vec{w}$

b. List 3 vectors in the span of $\vec{v}$ and $\vec{w}$.

c. For which values of h is the vector $\vec{u} = \begin{bmatrix} 2 \\ h \end{bmatrix}$ a linear combination of $\vec{v}$ and $\vec{w}$.

d. Write a system of equations that is equivalent to $x_1\vec{v} + x_2\vec{w} = \vec{0}$.

e. Write this system of equations as a matrix equation.

6. Compute each of the following. If the product is undefined, explain why.

a. $\begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

c. $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

7. Find the solution set to each of the following systems of equations. Give your answer in parametric form.

a. $\begin{array}{rcrcrcl} 2x_1 & + & x_2 & - & 2x_3 & = & 0 \\ x_1 & - & x_2 & - & x_3 & = & 0 \end{array}$ **b.** $\begin{array}{rcrcrcl} 2x_1 & + & x_2 & - & 2x_3 & = & 1 \\ x_1 & - & x_2 & - & x_3 & = & 2 \end{array}$

8. Determine whether the following set is linearly independent:

$$\begin{bmatrix} 1 \\ 0 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \\ 7 \\ 8 \end{bmatrix}$$

9. Determine whether the following set is linearly independent:

$$\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 9 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}$$

10. Consider the set of vectors $\begin{bmatrix} 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 4 \\ h \end{bmatrix}$

a. Determine a value of h for which this is a linearly independent set.

b. Determine a value of h for which this is a linearly dependent set.

11. Find the matrix that represents the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ with

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 2x_1 - x_3 \\ 0 \\ x_2 + x_3 \end{bmatrix}$$

12. Find the matrix that represents the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ that extends each vector by a factor of 3.

13. Find the matrix that represents the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ that first reflects across the x-axis, then reflects across the line $y = x$. Is there an easier way to describe this transformation?

14. Is the map $\mathbb{R}^2 \to \mathbb{R}^2$ given by $\begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} x+4 \\ y+4 \end{bmatrix}$ a linear transformation? Explain.

15. Is the map $\mathbb{R}^2 \to \mathbb{R}^2$ given by $\begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} -y \\ x \end{bmatrix}$ a linear transformation? Explain.