Math 303, Exam #1 Review **Fall 2008**

1. Find the distance between the points $(2, -1, 1)$ and $(1, 0, 3)$.

2. match each of the functions below with its graph.

a. $G(x, y) = \sin(x^2 + y^2)$

b. $f(x, y) = x^2 + y^2$

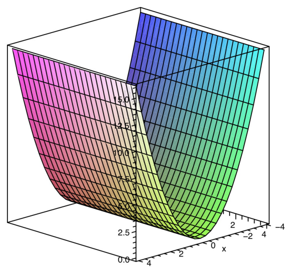
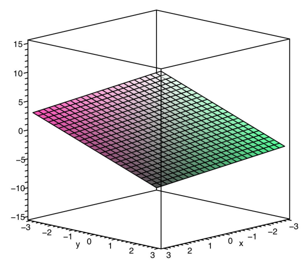
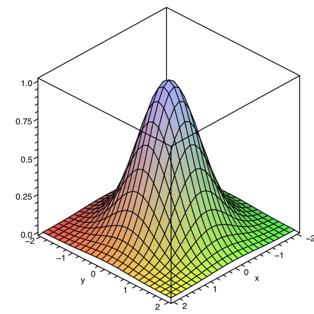
c. $g(x, y) = 3x + 2y$

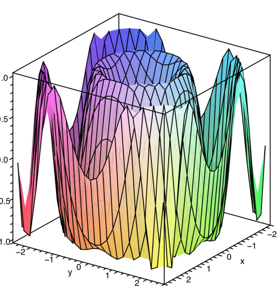
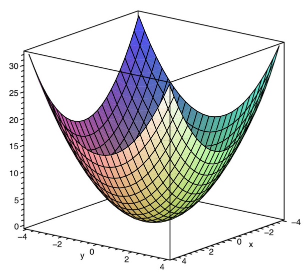
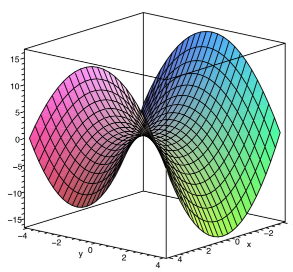
d. $h(x, y) = x^2 - y^2$

e. $k(x, y) = x^2$

f. $F(x, y) = \frac{1}{e^{x^2+y^2}}$

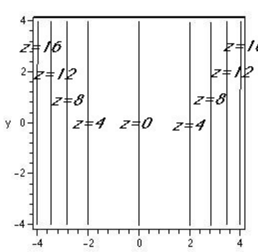
$k(x, y) = x^2$ $g(x, y) = 3x + 2y$ $F(x, y) = \frac{1}{e^{x^2+y^2}}$

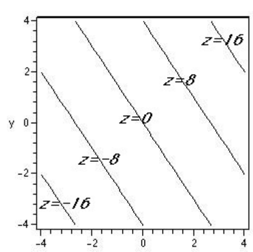




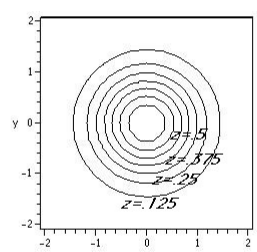




$G(x, y) = \sin(x^2 + y^2)$ $f(x, y) = x^2 + y^2$ $h(x, y) = x^2 - y^2$

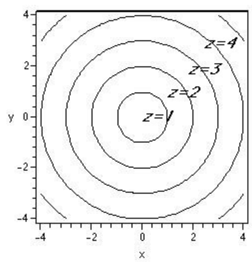
3. Make a contour diagram for each of the first three graphs above.
Contour diagram for $k(x, y) = x^2$, $g(x, y) = 3x + 2y$ and $F(x, y) = \frac{1}{e^{x^2+y^2}}$ in order are:







4. Make a contour diagram for $f(x, y) = \sqrt{x^2 + y^2}$.

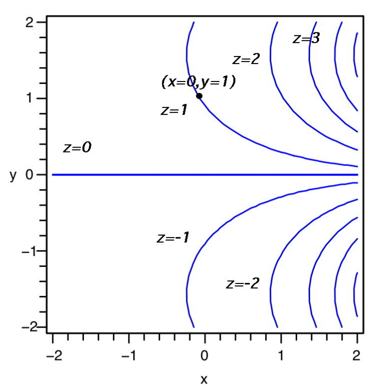


5. Describe the level surfaces of the function $f(x, y, z) = x^2 + y^2 + z$.
These are the surfaces $x^2 + y^2 + z = c$ for all real values of c. Solving for z, we have

$$z = c - (x^2 + y^2)$$

The surface $z = -(x^2 + y^2)$ is a down facing elliptic paraboloid, with vertex at the origin. Adding c shifts the paraboloid up by c (or down by $|c|$ for negative values of c.

6. Use the contour diagram for $f(x, y)$ below to approximate $f(0, 1)$.



$(0, 1)$ lies on the contour line $z = 1$, $\therefore$ $\boxed{f(0, 1) = 1}$.

7. Let $\vec{v} = 3\vec{i} - 2\vec{j} + \vec{k}$, $\vec{w} = \vec{i} - 2\vec{k}$, $\vec{u} = \vec{i} + \vec{j}$. Compute each of the following.

a. $\vec{v} - \vec{u}$. $\boxed{2\vec{i} - 3\vec{j} + \vec{k}}$

b. $\vec{v} \cdot \vec{w}$. 3-2=$\boxed{1}$

c. $\vec{v} \times \vec{w}$. $\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -2 & 1 \\ 1 & 0 & -2 \end{vmatrix} = \boxed{4\vec{i} + 7\vec{j} + 2\vec{k}}$

d. $||\vec{v} - \vec{w}||$. $\vec{v} - \vec{w} = 2\vec{i} - 2\vec{j} + 3\vec{k}$, so $\boxed{||\vec{v} - \vec{w}|| = \sqrt{17}}$.

e. $\vec{v}\cdot(\vec{v}+2\vec{w})$. $\vec{v}+2\vec{w}=5\vec{\imath}-2\vec{\jmath}-3\vec{k}$, so $\boxed{\vec{v}\cdot(\vec{v}+2\vec{w})=16}$

f. $\vec{u}\cdot(\vec{v}\times\vec{w})$. $\vec{u}\cdot(4\vec{\imath}+7\vec{\jmath}+2\vec{k})=4+7=\boxed{11}$

g. Find the angle between $\vec{v}$ and $\vec{w}$.
$1=\vec{v}\cdot\vec{w}=||\vec{v}||\,||\vec{w}||\cos\theta=\sqrt{14}\sqrt{5}\cos\theta$, so $\cos\theta=\frac{1}{\sqrt{70}}$, $\theta=\cos^{-1}\left(\frac{1}{\sqrt{70}}\right)\approx\boxed{1.4510\text{ radians}}$

8. Find the equation for the plane with normal vector $\vec{n}=\frac{1}{2}\,\vec{\imath}+\frac{1}{2}\,\vec{\jmath}+\frac{1}{\sqrt{2}}\,\vec{k}$ containing the point $(1,-2,0)$.

$0=\frac{1}{2}x+\frac{1}{2}y+\frac{1}{\sqrt{2}}z+d$. Solving for d:
$0=\frac{1}{2}\cdot 1+\frac{1}{2}(-2)+\frac{1}{\sqrt{2}}\cdot 0+d=-\frac{1}{2}+d$, so $d=\frac{1}{2}$.

$$\boxed{0=\frac{1}{2}x+\frac{1}{2}y+\frac{1}{\sqrt{2}}z+\frac{1}{2}}$$