Math 303, Exam #1 **Name:**________________________________

Fall 2002

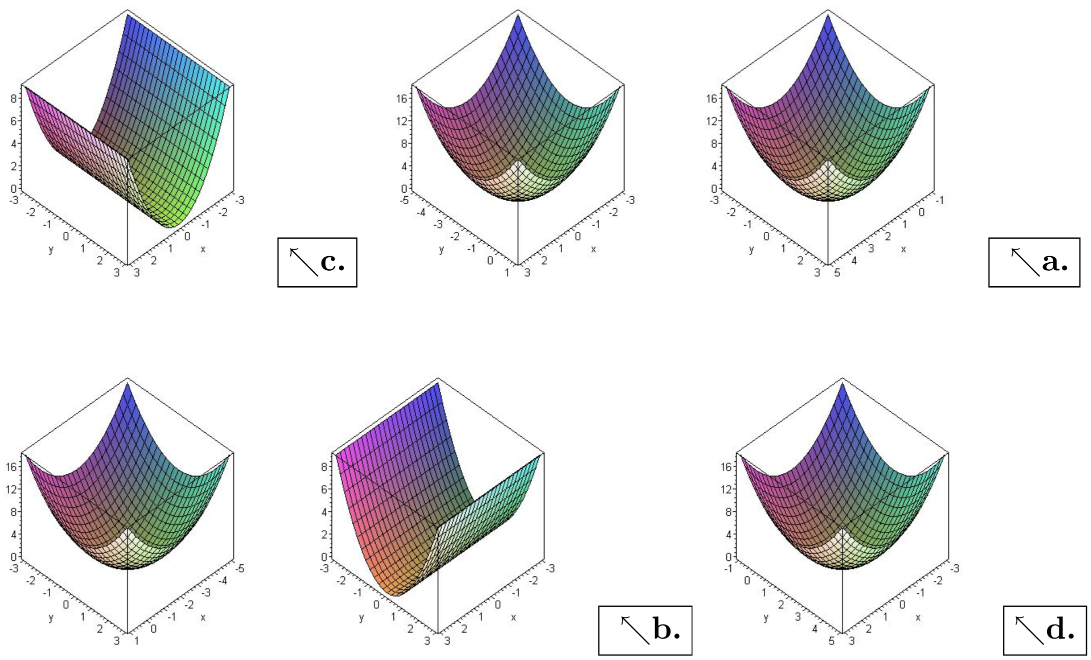
1. Match each of the functions below with its graph.

a. $f(x, y) = (x - 2)^2 + y^2$

b. $g(x, y) = y^2$

c. $h(x, y) = x^2$

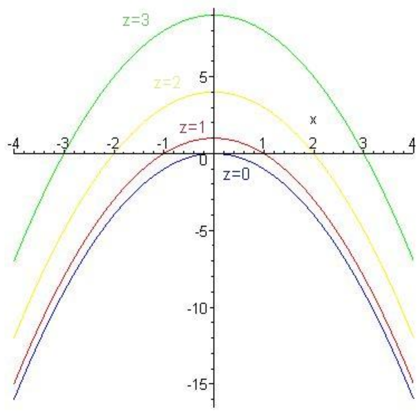
d. $k(x, y) = x^2 + (y - 2)^2$



2. Find the distance between the points $(-1, 1, 0)$, and $(0, -1, 3)$.

$$d = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$$

3. Make a contour diagram for $f(x,y) = \sqrt{x^2+y}$. Include 4 level curves. Label the axes and each level curve.



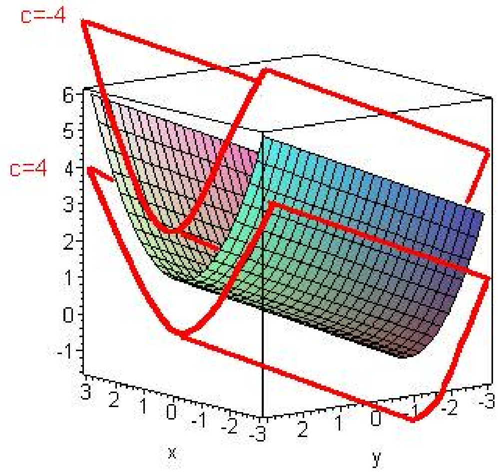
4. Let $f(x,y,z) = x^2 + y - 2z$.

a. Find the equations for the level surfaces for $c = -4$, $c = 0$ and $c = 4$.

$x^2 + y - 2z = c \Longrightarrow z = \frac{1}{2}x^2 + \frac{1}{2}y - \frac{1}{2}c.$

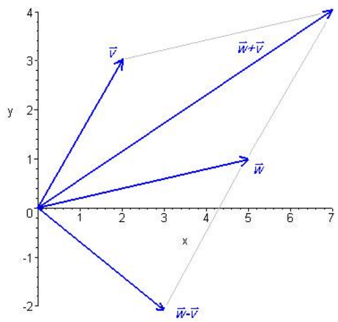
$$\begin{aligned} c = 0: \;\; z &= \frac{1}{2}x^2 + \frac{1}{2}y \\ c = -4: \;\; z &= \frac{1}{2}x^2 + \frac{1}{2}y + 2 \\ c = 4: \;\; z &= \frac{1}{2}x^2 + \frac{1}{2}y - 2 \end{aligned}$$

b. The graph below is a level surface for $f(x, y, z)$ with $c = 0$. On the same graph, sketch the level surfaces for $c = -4$, and $c = 4$. Be careful about scale, and be sure to label each surface.



5. Let $\vec{v} = 2\vec{i} + 3\vec{j}$, and $\vec{w} = 5\vec{i} + \vec{j}$.

a. Sketch $\vec{v}$, $\vec{w}$, $\vec{w} - \vec{v}$ and $\vec{v} + \vec{w}$.



b. Find the angle θ between $\vec{v}$ and $\vec{w}$.

First compute: $\vec{v} \cdot \vec{w} = 2 \cdot 5 + 3 \cdot 1 = 13$, $\|\vec{v}\| = \sqrt{4+9} = \sqrt{13}$, and $\|\vec{w}\| = \sqrt{25+1} = \sqrt{26}$.

Now,

$$\begin{aligned} \vec{v} \cdot \vec{w} &= \|\vec{v}\|\|\vec{w}\| \cos\theta \\ 13 &= \sqrt{13}\sqrt{26}\cos\theta \\ &= 13\sqrt{2}\cos\theta \\ \cos\theta &= \frac{1}{\sqrt{2}} \\ \theta &= \frac{\pi}{4} \end{aligned}$$

c. Compute the area of the parallelogram determined by $\vec{v}$ and $\vec{w}$.

$$\text{Area}=\|\vec{v}\|\|\vec{w}\| \sin\theta = \sqrt{13}\sqrt{26}\sin\frac{\pi}{4} = 13\sqrt{2} \cdot \frac{\sqrt{2}}{2} = 13.$$

6. Resolve the vector that starts at $(-1, 1, 0)$ and ends at $(0, -1, 3)$.

$$(0 - (-1))\vec{i} + (-1 - 1)\vec{j} + (3 - 0)\vec{k} = \vec{i} - 2\vec{j} + 3\vec{k}.$$

7. Let $\vec{v} = 2\vec{i} - \vec{j}$, $\vec{w} = \vec{i} - \vec{j} + \vec{k}$ and $\vec{u} = \vec{i} + \vec{j}$.

a. Compute $\vec{v} \cdot \vec{w}$.

$$\vec{v} \cdot \vec{w} = 2 + (-1)(-1) = 3$$

b. Compute $3(\vec{v} - \vec{u}) \cdot \vec{w}$.

$$\begin{aligned} 3(\vec{v} - \vec{u}) \cdot \vec{w} &= 3(\vec{i} - 2\vec{j}) \cdot (\vec{i} - \vec{j} + \vec{k}) \\ &= 3(1 + 2) \\ &= 9 \end{aligned}$$

c. Compute $\vec{v} \times \vec{u}$.

$$\begin{aligned} \vec{v} \times \vec{u} &= (2\vec{i} - \vec{j}) \times (\vec{i} + \vec{j}) \\ &= 2\vec{i} \times \vec{i} + 2\vec{i} \times \vec{j} - \vec{j} \times \vec{i} - \vec{j} \times \vec{j} \\ &= 2\vec{i} \times \vec{j} - \vec{j} \times \vec{i} \\ &= 3\vec{i} \times \vec{j} \\ &= 3\vec{k} \end{aligned}$$

8. Find the equation for the plane containing the points $(1, 2, 0)$, $(-1, 1, 0)$, and $(0, -1, 3)$.

Start by finding the vector $\vec{v}$ from $(1, 2, 0)$ to $(-1, 1, 0)$:

$$\vec{v} = (-1 - 1)\vec{\imath} + (1 - 2)\vec{\jmath} + (0 - 0)\vec{k} = -2\vec{\imath} - \vec{\jmath},$$

and the vector $\vec{w}$ from $(1, 2, 0)$ to $(0, -1, 3)$:

$$\vec{w} = (0 - 1)\vec{\imath} + (-1 - 2)\vec{\jmath} + (3 - 0)\vec{k} = -\vec{\imath} - 3\vec{\jmath} + 3\vec{k}$$

Find a vector which is normal to the plane spanned by $\vec{v}$ and $\vec{w}$ by computing $\vec{v} \times \vec{w}$:

$$\vec{n} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ -2 & -1 & 0 \\ -1 & -3 & 3 \end{vmatrix} = (-3)\vec{\imath} - (-6)\vec{\jmath} + (6 - 1)\vec{k} = -3\vec{\imath} + 6\vec{\jmath} + 5\vec{k}$$

The plane we want is normal to $\vec{n}$ and contains the point $(1, 2, 0)$:

$$\boxed{-3(x - 1) + 6(y - 2) + 5(z - 0) = 0}$$