

Math 203, Exam #2
Fall 2008

Name: _____

Please write out the full pledge and sign below.

Please show all your work and explain your answers on each problem.

1. True/False Decide whether each of the following statements is true or false. **Please give a short explanation for your answer.**

a. **T F** The map $\mathbb{R}^2 \rightarrow \mathbb{R}^2$ given by $\begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} 2x + 1 \\ y \end{bmatrix}$ is a linear transformation.

b. **T F** If A is an $n \times n$ matrix and the columns of A span \mathbb{R}^n , then the columns of A are linearly independent.

c. **T F** If A is an $n \times n$ matrix and the equation $A\vec{x} = \vec{0}$ has a nontrivial solution, then A has fewer than n pivot positions.

2. Let $\vec{v}_1 = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$.

a. Give an example of a vector \vec{v}_3 so that the set $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly independent.

b. Give an example of a vector \vec{v}_3 so that the set $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly dependent.

3. Find the matrix for the transformation of \mathbb{R}^2 that first rotates counterclockwise by $\frac{\pi}{4}$ radians about the origin, then reflects across the line $y = x$.

4. Find the 3×3 matrix that produces the following transformation of \mathbb{R}^2 using homogeneous coordinates: First reflect across the x -axis, then shift up by 2.

5. Suppose a transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ is represented by the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 0 & 2 & 8 \\ 0 & 0 & 0 \end{bmatrix}$.

Express your answers to the following in simplest form.

a. Find the null space of T .

b. Find the image of T .

6. Solve the following system of equations by using the inverse of an appropriate matrix.

$$\begin{array}{rcl} x_1 & - & 2x_2 = 1 \\ 3x_1 & - & x_2 = 2 \end{array}$$

7. Let A be an $n \times n$ matrix. Give 3 statements that are equivalent to the statement: “ A is an invertible matrix.”

8. Which of the following matrices are invertible? Explain your answer using as few calculations as possible.

a.
$$\begin{bmatrix} 1 & 3 & 0 \\ 2 & 0 & 4 \\ 3 & 3 & 4 \end{bmatrix}$$

b.
$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 0 & 2 \\ 0 & 1 & -3 \end{bmatrix}$$

9. Let $B = \begin{bmatrix} 2 & 1 & -1 \\ 4 & -4 & 8 \\ -6 & 3 & -8 \end{bmatrix}$

- a. Find an LU factorization of B

b. Use your factorization to solve the matrix equation $B\mathbf{x} = \begin{bmatrix} 3 \\ 2 \\ -4 \end{bmatrix}$

10. Determine whether the vector $\vec{b} = \begin{bmatrix} 8 \\ 3 \\ -4 \end{bmatrix}$ is in the subspace of \mathbb{R}^3 spanned by

$$\vec{v}_1 = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} \text{ and } \vec{v}_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}.$$

11. Give an example of a basis for \mathbb{R}^3 other than the standard basis.

12. Let $A = \begin{bmatrix} -2 & 0 & 0 \\ 1 & 1 & 4 \\ 4 & 0 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 9 & 0 \\ 1 & -8 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 4 \\ 4 & 2 & 8 \end{bmatrix}$

Compute the following, or explain why the expression is undefined.

a. AB

b. BA

c. The determinant of C .

d. C^{-1}

e. A^{-1}