Math 203, Exam #2 Review **Fall 2009**

Your exam will cover sections 1.9, 2.1-2.3, 2.5-2.7. In addition to these problems, you should look over your homework - including the true/false questions. Be prepared to justify your answers. Topics we've studied include: Linear Independence, Linear Transformations, Matrix Algebra, the Inverse of a Matrix, *LU* Factorization, Homogeneous Coordinates.

1. Determine whether the following set is linearly independent:

$$\begin{bmatrix} 1 \\ 0 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \\ 7 \\ 8 \end{bmatrix}$$

2. Determine whether the following set is linearly independent:

$$\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 9 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}$$

3. Consider the set of vectors $\begin{bmatrix} 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 4 \\ h \end{bmatrix}$

a. Determine a value of h for which this is a linearly independent set.

b. Determine a value of h for which this is a linearly dependent set.

4. Find the matrix that represents the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ that rotates each vector $\frac{\pi}{3}$ radians counterclockwise about the origin.

5. Find the matrix that represents the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ with

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 2x_1 - x_3 \\ 0 \\ x_2 + x_3 \end{bmatrix}$$

6. Find the matrix that represents the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ that extends each vector by a factor of 3.

7. Find the matrix that represents the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ that first reflects across the x-axis, then reflects across the line $y = x$. Is there an easier way to describe this transformation?

8. Is the map $\mathbb{R}^2 \to \mathbb{R}^2$ given by $\begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} x+4 \\ y+4 \end{bmatrix}$ a linear transformation? Explain.

9. Is the map $\mathbb{R}^2 \to \mathbb{R}^2$ given by $\begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} -y \\ x \end{bmatrix}$ a linear transformation? Explain.

10. Suppose a transformation $T : \mathbb{R}^4 \to \mathbb{R}^3$ is represented by the matrix $A = \begin{bmatrix} 1 & 2 & -3 & 3 \\ 0 & 1 & 4 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$.

a. Find all solutions to the eqaution $T\vec{x} = \vec{0}$

b. Find the image of T.

11. Suppose a transformation $T : \mathbb{R}^3 \to \mathbb{R}^4$ is represented by the matrix $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$.

a. Find all solutions to the eqaution $T\vec{x} = \vec{0}$

b. Find the image of T.

12. Let $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -2 & -2 \\ 4 & 0 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 4 & 2 \\ 0 & -1 \\ 2 & 3 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 3 \\ -2 & 3 \\ 0 & 1 \end{bmatrix}$

Compute the following, or explain why the expression is undefined.
AB, BA, AC, BC, A^{-1}, B^{-1}, A^T, $C - 2D$, $A^2 + 2A + I$.

13. Solve the following system of equations by using the inverse of an appropriate matrix.

$$\begin{array}{rcrcl} 2x_1 & + & x_3 & = & 4 \\ x_1 & - & 2x_3 & = & 4 \end{array}$$

14. Let A be an $n \times n$ matrix. Give 4 statements that are equivalent to the statement "A is an invertible matrix."

15. Which of the following matrices are invertible? Use as few calculations as possible.
$\begin{bmatrix} 1 & 3 & 3 \\ -2 & 4 & 4 \\ 0 & 1 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 & 0 \\ -2 & 3 & 0 \\ 5 & 1 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & 3 & 3 \\ 0 & 3 & 3 \\ 0 & 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & 3 & 3 \\ 0 & 2 & 8 \\ 0 & 1 & 4 \end{bmatrix}$

16. Find an LU factorization for each of the following matrices:
$A = \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 3 & -5 \\ -4 & -5 & 7 \\ 8 & 6 & -8 \end{bmatrix}$

Use your factorization to solve the matrix equation $B\mathbf{x} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

17. Find the 3×3 matrix that produces the following transformation of $\mathbb{R}^2$ using homogeneous coordinates.

Scale the x coordinate by a factor of 2, then shift right 1 and up 3.

18. Find the determinant of the matrix $A = \begin{bmatrix} 4 & 3 \\ -4 & -5 \end{bmatrix}$.