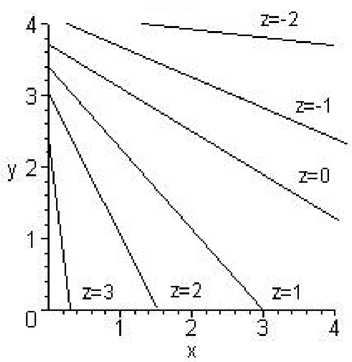
Math 303, Exam #2 **Name: Solutions**
Fall 2002

1. The contour diagram for $f(x, y)$ is drawn below.



a. Approximate $f_x(1, 1)$.

$\boxed{f_x(1, 1) \approx -1}$

b. Approximate $f_y(1, 1)$.

$\boxed{f_y(1, 1) \approx -.77}$

c. Sketch grad $f(1, 1)$ on the contour diagram.

d. Is $f_{xx}(1, 1)$ positive or negative?

At the point (1,1) f_x is increasing as you increase x, so $\boxed{f_{xx}(1, 1) > 0}$

2. Let $f(x, y) = 2x^2 - xy$, and $\vec{v} = 2\vec{\imath} - \vec{\jmath}$.

a. Compute $f_x(x, y)$.

$\boxed{f_x(x, y) = 4x - y}$

b. Compute $f_y(x, y)$.

$\boxed{f_y(x, y) = -x}$

c. Compute $f_{xx}(x, y)$.

$\boxed{f_{xy}(x, y) = 4}$

d. Compute $f_{xy}(x, y)$.

$\boxed{f_{xy}(x, y) = -1}$

e. Compute the directional derivative of $f(x, y)$ in the direction of $\vec{v}$, at the point $(2, 0)$.

$$\boxed{f_{\vec{v}}(2, 0) = \frac{18}{\sqrt{5}}}$$

3. Compute the regression line (line of best fit) for the points $(0, 1)$, $(1, 3)$, $(3, 6)$. In order to receive full credit, you must use the methods discussed in this class, and show all your work.

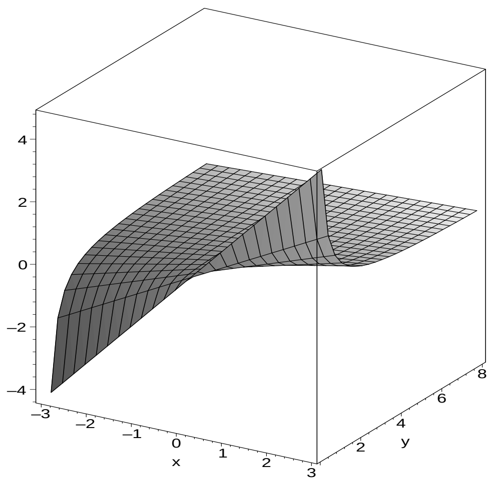
$$\boxed{y = \frac{23}{14}x + \frac{8}{7}}$$

4. Let $f(x, y, z) = 4yz - 2x - 2y$.

a. Compute $\operatorname{grad} f(3, 2, 1)$.

$$\boxed{\operatorname{grad} f(3, 2, 1) = -2\vec{\imath} + 2\vec{\jmath} + 8\vec{k}}$$

b. The level curve for $f = 0$ is drawn below. Sketch $\operatorname{grad} f(3, 2, 1)$ on the the graph.



5. Let $g(x, y) = 2\sqrt{xy}$, $x(u, v) = u - v$, $y(u, v) = u + v$. Use the chain rule to compute the partial derivative g_v. Give your answers in terms of u and v.

$$\boxed{g_v = -\frac{\sqrt{y}}{\sqrt{x}} + \frac{\sqrt{x}}{\sqrt{y}} = \frac{x}{\sqrt{xy}} - \frac{y}{\sqrt{xy}} = \frac{x - y}{\sqrt{xy}}}$$

6. Let $H(x) = e^{xy} - x$.

a. Find the Taylor polynomial of degree 2 for H near $(-1, 0)$.

$$\boxed{T = 2 - (x + 1) - y + \frac{1}{2}y^2 + (x + 1)y}$$

b. Use your answer to part **a** to approximate $H(-1.1, .2)$.

$$\boxed{H(-1.1, .2) \approx T(-1.1, .2) = 1.94}$$

7. Let $p(x, y) = x^2 - 2xy - 3y^2$. Find all of the critical points. For each critical point decide if it is a local maximum, local minimum, saddle point or none of these.

The only critical point is (0,0), and since $D(0, 0) = -40 < 0$, $\boxed{\text{(0,0) is a saddle point.}}$

8. Let $F(x, y) = x^4 - 2x^2y + 2y$. Find all of the critical points. For each critical point decide if it is a local maximum, local minimum, saddle point or none of these.

The critical points are $\boxed{(1, 1), (-1, 1)}$ and they are $\boxed{\text{both saddle points}}$

9. Use Lagrange multipliers to find the maximum and minimum values of $f(x, y) = xy^2$. subject to the constraint $g(x, y) = 3x - 4y \leq 5$.

There is a line of critical points at $y = 0$. Lagrange multipliers yields the points $(\frac{5}{3}, 0)$ and $(\frac{5}{6}, -\frac{5}{8})$. However, since the constraint inequality does not determine a closed, bounded region we have the possibility that there is no solution. In this case, there is no solution, since $f \to \infty$ when x is positive and $f \to -\infty$ when x is negative.