Math 203, Exam #3 **Name:**______________________________

Fall 2009

Please write out the full pledge and sign below.

Please show all your work and explain your answers on each problem.

1. True/False Decide whether each of the following statements is true or false. **Please give a short explanation for your answer.**

a. T F The row space of A is the domain of the map $\vec{x} \mapsto A\vec{x}$.

b. T F If A is an $n \times m$ matrix, then $\text{Col}(A) \subseteq \mathbb{R}^n$.

c. T F

2. Determine which of the following are bases for the indicated vector space. Give a short explanation for your answer.

a. $\left\{ \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \end{bmatrix}, \begin{bmatrix} 5 \\ 4 \end{bmatrix} \right\}$, for $\mathbb{R}^2$

b. $\left\{ \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix} \right\}$, for $\mathbb{R}^3$

3. Suppose $A = \begin{bmatrix} B & I_2 \\ 0 & C \end{bmatrix}$, where B and C are 2×2 matrices, and I_2 is the 2×2 identity matrix. Find a formula for A^{-1} in terms of B and C. Show all your work.

a. Use your answer above to find the inverse of the matrix $A = \begin{bmatrix} 1 & 3 & 1 & 0 \\ 1 & 4 & 0 & 1 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 1 & 2 \end{bmatrix}$

4. Let $\mathcal{B} = \left\{ \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$.

a. Show that $\mathcal{B}$ is a basis for $\mathbb{R}^3$.

b. Find $[\vec{x}]_{\mathcal{B}}$, where $\vec{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

c. Find $\vec{x}$, where $[\vec{x}]_{\mathcal{B}} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

5. Let A be an $n \times n$ matrix. Give three of the new statements that are equivalent to the statement A is an invertible matrix.

6. Suppose $\det \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = 11$. Compute the determinant of each of the following.

a. $\begin{pmatrix} g & h & i \\ a & b & c \\ d & e & f \end{pmatrix}$

b. $\begin{pmatrix} 2a+g & 2b+h & 2c+i \\ -d & -e & -f \\ g & h & i \end{pmatrix}$

7. Recall that $\mathbb{P}_2$ is the vector space consisting of polynomials of degree at most 2.

a. What is the dimension of $\mathbb{P}_2$?

b. Find a basis for $\mathbb{P}_3$ other than the standard basis.

c. Find $[3t^2 + t - 2]_{\mathcal{B}}$ with respect to this basis.

8. Let $A = \begin{pmatrix} 3 & 9 & 12 & 6 & 12 & 3 \\ 2 & 6 & 10 & 5 & 9 & 1 \\ 1 & 3 & 8 & 5 & 8 & -1 \\ 3 & 9 & 16 & 7 & 12 & 2 \\ -2 & -6 & -4 & -1 & -4 & -2 \\ 5 & 15 & 24 & 13 & 24 & 2 \end{pmatrix}$.

Find bases for each of the following spaces:

a. Null(A)

b. Col(A)

c. Row(A)

Find the dimension of each of the following spaces:

d. Null(A)

e. Col(A)

9. A particular strain of flu is going around campus. There are students who have never had the flu, students who have the flu now, and students who are immune - either by getting a flu shot or by recovering from the flu. Each week, 4% of the students who never had the flu will get the flu, and 20% of the students who never had flu will get a flu shot, and become immune. Also 25% of the student with flu will recover and become immune.

a. Set up the stochastic matrix for this situation.

b. If we start with 90% of the students never having had the flu, and 10% having the flu now, what will the situation be after 4 weeks assuming the trend continues?