

**Math 203, Exam #3**  
**Fall 2008**

Name: \_\_\_\_\_

Please write out the full pledge and sign below.

Please show all your work and explain your answers on each problem.

**1. True/False** Decide whether each of the following statements is true or false. **Please give a short explanation for your answer.**

a. **T F** The column space of  $A$  is the range of the map  $\vec{x} \mapsto A\vec{x}$ .

b. **T F** If  $A$  is an  $n \times m$  matrix, then  $\text{Col}(A) = \mathbb{R}^n$ .

c. **T F** If  $A$  is a matrix, then the dimension of  $\text{Row}(A)$  is the number of pivots of  $A$ .

**2.** Let  $\mathcal{B} = \left\{ \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \begin{bmatrix} -5 \\ 3 \end{bmatrix} \right\}$ .

a. Explain how you can tell that  $\mathcal{B}$  is a basis for  $\mathbb{R}^2$ .

b. Find  $[\vec{x}]_{\mathcal{B}}$ , where  $\vec{x} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$ .

c. Find  $\vec{x}$ , where  $[\vec{x}]_{\mathcal{B}} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$ .

3. Let  $A$  be an  $n \times n$  matrix. Give two of the new statements that are equivalent to the statement  $A$  is an invertible matrix.

4. Suppose

$$\det \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = 4, \det \begin{pmatrix} b & c \\ h & i \end{pmatrix} = 0, \det \begin{pmatrix} a & c \\ g & i \end{pmatrix} = -7 \text{ and } \det \begin{pmatrix} a & b \\ g & h \end{pmatrix} = 2.$$

Compute the determinant of each of the following.

a.  $\begin{pmatrix} d & e & f \\ g & h & i \\ a & b & c \end{pmatrix}$

b.  $\begin{pmatrix} 0 & a & b & c \\ 2 & 0 & -4 & 1 \\ -1 & d & e & f \\ 0 & g & h & i \end{pmatrix}$

5. Recall that  $\mathbb{P}_3$  is the vector space consisting of polynomials of degree at most 3.

a. Find a basis for  $\mathbb{P}_3$  that contains the polynomial  $1 - t^2$ .

b. Find  $[2 + 3t + t^3]_{\mathcal{B}}$  with respect to this basis.

6. Let  $A = \begin{pmatrix} 1 & 2 & 0 & 3 & -1 & 4 \\ -2 & -4 & -3 & -4 & 3 & -8 \\ 1 & 2 & -6 & 3 & -1 & 7 \\ 2 & 4 & 9 & 4 & -3 & 4 \\ 1 & 2 & -6 & 11 & 3 & 0 \end{pmatrix}$ .

Find bases for each of the following spaces:

a.  $\text{Null}(A)$

b.  $\text{Col}(A)$

c.  $\text{Row}(A)$

Find the dimension of each of the following spaces:

d.  $\text{Null}(A)$

e.  $\text{Col}(A)$

f.  $\text{Row}(A)$

7. Determine which of the following are bases for the indicated vector space. Give a short explanation for your answer.

a.  $\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$ , for  $\mathbb{R}^2$

b.  $\{(0, 1, 0), (0, 0, 1)\}$ , for  $\text{Row} \begin{bmatrix} 0 & 4 & 7 \\ 0 & -1 & 2 \end{bmatrix}$

c.  $\left\{ \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$ , for  $\mathbb{R}^3$

d.  $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ , for  $\mathbb{R}^3$