

**Math 303, Exam #3**  
**Fall 2002**

**Name:** \_\_\_\_\_

Please write out the full pledge, and sign below.

1. Use the following table of values to find a lower estimate of  $\int_R f(x, y) dA$ , where  $R$  is the rectangle  $0 \leq x \leq 1$ ,  $-1 \leq y \leq 0$ .

		$x$			
		0	.5	1	1.5
$y$	-1	1	5	7	10
	-.5	2	4	8	11
	0	3	5	9	13
	.5	4	6	11	15

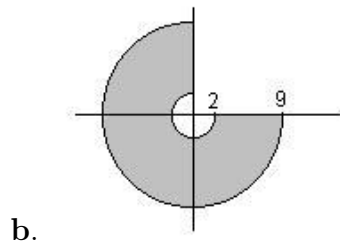
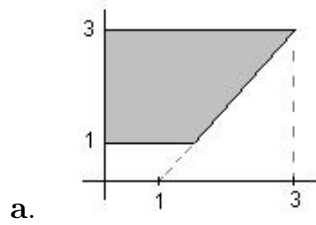
Compute each of the following integrals.

2.  $\int_0^1 \int_{-1}^1 (2x^2 + y^2) dx dy$

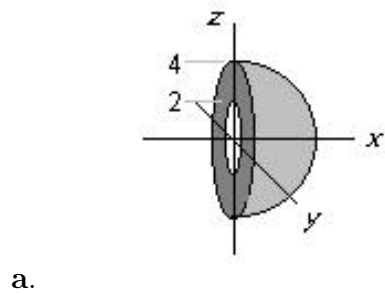
3.  $\int_0^1 \int_y^{\sqrt{y}} (2xy - y^2) dx dy$

4. Sketch the region of integration for the integrals in each of the last two problems.

5. For each of the regions, write the integral of  $f(x, y) = x^2 - y^2$  over the region as an iterated integral. State which coordinate system you are using.



6. For each of the following regions, set up a triple integral of  $f(x, y, z) = xy + z$  over the region.



b. The cylinder of radius 2 and height 4, centered at the origin.

7. Parametrize the line from the point  $(2,4,-1)$  to the point  $(0,1,2)$ .

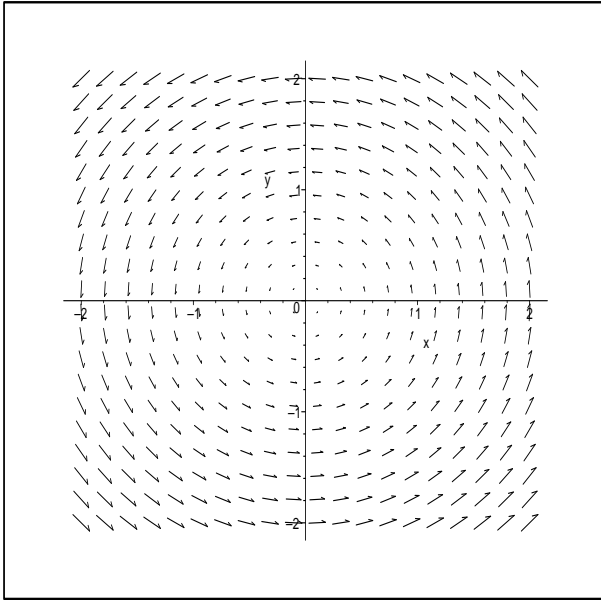
8. Find  $a$  and  $b$  so that the following lines intersect.

$$\begin{aligned}\vec{r}_1(t) &= (t-1)\vec{i} + (2t+1)\vec{j} + (3-t)\vec{k} \\ \vec{r}_2(t) &= (t+2)\vec{i} + (1-t)\vec{j} + (at+b)\vec{k}\end{aligned}$$

9. Sketch the vector field  $\vec{F} = y\vec{i}$

10. Parametrize the plane through the point  $(1, -1, 2)$  that contains the vectors  $2\vec{i} - \vec{j} + \vec{k}$  and  $\vec{i} + 3\vec{j} - \vec{k}$

11. The vector field  $\vec{F}(x, y) = -y\vec{i} + x\vec{j}$  is drawn below.



a. determine which of the following parametrized curves could be flow lines for this vector field.

- (i)  $\vec{r}_1(t) = 2 \cos(t)\vec{i} - 2 \sin(t)\vec{j}$
- (ii)  $\vec{r}_2(t) = 2 \cos(t)\vec{i} + 2 \sin(t)\vec{j}$
- (iii)  $\vec{r}_3(t) = -2 \cos(t)\vec{i} + \sin(t)\vec{j}$
- (iv)  $\vec{r}_4(t) = \cos(t)\vec{i} - \sin(t)\vec{j}$
- (v)  $\vec{r}_5(t) = \sin(t)\vec{i} - \cos(t)\vec{j}$

b. Write down the differential equations associated to the vector field. For each of the the curves that could be flow lines, show that the curve satisfies the differential equations.