Math 203, Exam #3 Review **Fall 2009**

Your exam will cover §2.4, §2.7-2.8, §3.1, 3.2 and §4.1-4.9 In addition to these problems, you should look over your homework - including the true/false questions. Be prepared to justify your answers. Topics we've studied include: Subspaces of $\mathbb{R}^n$, Determinants, Vector Spaces, Subspaces,Linear Independence, Spanning set, Bases, Linear Transformations, Null Space, Column Space, Row Space, Change of coordinates, Dimension, Rank, Markov Chains.

1. Suppose $A = \begin{bmatrix} B & C \\ 0 & D \end{bmatrix}$, Where B, C and D are 3×3 matrices. Find a formula for A^{-1}. Are there any conditions on B, C and D?

2. Suppose a transformation $T : \mathbb{R}^4 \to \mathbb{R}^3$ is represented by the matrix $A = \begin{bmatrix} 1 & 2 & -3 & 3 \\ 0 & 1 & 4 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$.

a. Find the null space of T.

b. Find the image of T.

3. Suppose a transformation $T : \mathbb{R}^3 \to \mathbb{R}^4$ is represented by the matrix $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$.

a. Find the null space of T.

b. Find the image of T.

4. Let $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -2 & -2 \\ 4 & 0 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 4 & 2 \\ 0 & -1 \\ 2 & 3 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 3 \\ -2 & 3 \\ 0 & 1 \end{bmatrix}$

5. Which of the following matrices are invertible? Use as few calculations as possible.
$\begin{bmatrix} 1 & 3 & 3 \\ -2 & 4 & 4 \\ 0 & 1 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 & 0 \\ -2 & 3 & 0 \\ 5 & 1 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & 3 & 3 \\ 0 & 3 & 3 \\ 0 & 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & 3 & 3 \\ 0 & 2 & 8 \\ 0 & 1 & 4 \end{bmatrix}$

6. Draw an example of a subspace of $\mathbb{R}^2$.

7. Draw an example of a subset of $\mathbb{R}^2$ that is not a subspace.
$\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 4 \end{bmatrix}$.

8. Give an example of a basis for $\mathbb{R}^2$ other than the standard basis. Do the same for $\mathbb{R}^3$. Give an example of a pair of vectors in $\mathbb{R}^2$ that do not form a basis for $\mathbb{R}^2$.

9. Find the determinant of the matrices $A = \begin{bmatrix} 4 & 3 \\ -4 & -5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & 4 \\ -1 & 0 & 0 \end{bmatrix}$.

10. Suppose $\det \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = 12$, $\det \begin{pmatrix} b & c \\ h & i \end{pmatrix} = 5$, $\det \begin{pmatrix} a & c \\ g & i \end{pmatrix} = 9$ and $\det \begin{pmatrix} a & b \\ g & h \end{pmatrix} =$

3. Compute the determinant of each of the following.

a. $\begin{pmatrix} g & h & i \\ a & b & c \\ d & e & f \end{pmatrix}$

b. $\begin{pmatrix} a & b & c \\ 2a & 2b & 2c \\ d & e & f \end{pmatrix}$

c. $\begin{pmatrix} a & b & c \\ 2d & 2e & 2f \\ g & h & i \end{pmatrix}$

d. $\begin{pmatrix} a+d & b+e & c+f \\ d & e & f \\ g & h & i \end{pmatrix}$

e. $\begin{pmatrix} 3 & 2 & 1 & 0 \\ 0 & a & b & c \\ 0 & d & e & f \\ 0 & g & h & i \end{pmatrix}$

f. $\begin{pmatrix} 3 & 2 & 1 & 0 \\ 0 & a & b & c \\ 1 & d & e & f \\ 0 & g & h & i \end{pmatrix}$

11. Determine which of the following are subspaces of $\mathbb{P}_6$.

a. $S = \{at^2 \,|\, a \in \mathbb{R}\}$

b. $T = \{a + t^2 \,|\, a \in \mathbb{R}\}$

c. $U = \{p(t) \in \mathbb{P}_6 \,|\, p(0) = 0\}$

d. The set, V, of polynomials of even degree at most 6.

12. Find a basis for $\mathbb{P}_3$ other than the standard basis. Find $[2 + 3t + t^3]_{\mathcal{B}}$ with respect to this basis.

13. Find a basis for the subpace of $\mathbb{R}^4$ consisting of vectors of the form $\begin{bmatrix} c - 2d \\ 2d \\ c \\ d \end{bmatrix}$, $c, d \in \mathbb{R}$.

14. Let $A = \begin{pmatrix} 4 & 6 & 14 & 0 & 2 \\ 2 & 3 & 10 & -1 & 0 \\ 6 & 9 & 18 & 1 & 10 \\ 2 & 3 & 4 & 1 & 8 \end{pmatrix}$. Find bases for and compute the dimension of each of the following spaces:

a. Null(A)

b. Col(A)

c. Row(A)

15. Find a matrix M so that $\text{Col}(M) = \left\{ \begin{bmatrix} a+b \\ b+c-d \\ a+2b-c \end{bmatrix} \middle| \, a, b, c, d \in \mathbb{R} \right\}$

16. Determine which of the following are bases for $\mathbb{R}^3$. Give a short explanation for your answer.

a. $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}$

b. $\left\{ \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

c. $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$

d. $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$

17. Let $\mathcal{B} = \left\{ \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -4 \\ 2 \end{bmatrix} \right\}$.

a. Show that $\mathcal{B}$ is a basis for $\mathbb{R}^3$.

b. Find $[\vec{x}]_{\mathcal{B}}$, where $\vec{x} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$.

c. Find $\vec{x}$, where $[\vec{x}]_{\mathcal{B}} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$.

18. Let A be an $n \times n$ matrix. Give three of the new statements that are equivalent to the statement A is an invertible matrix.

19. A particular strain of flu is going around campus. There are students who have never had the flu, students who have the flu now, and students who are recovered from the flu and now immune. Each week, 5% of the students who never had the flu will get the flu and 20% of the student with flu will recover and become immune.

a. Set up the stochastic matrix for this situation.

b. If we start with 80% of the students never having had the flu, and 20% having the flu now, what will the situation be after 10 weeks assuming the trend continues?

c. Is there a steady state for this situation? If so, find it.

20. Each month, 10% of the employed people lose their job, and half of the unemployed people get a job.

a. Set up the stochastic matrix for this situation.

b. Starting with 7% unemployment, what will the situation be in one year, assuming this

trend continues?