Math 303, Exam #3 Review **Fall 2008**

The exam will cover §17.1-17.5 and §18.1-18.3. In addition to these problems, you will want to review worksheets 12-16 (Note: Worksheet 12 was mislabeled as Worksheet 11. It is the one about parametrizing curves). For the exam, you will be responsible for following tasks using Maple: plotting a parametrized curve, plotting a vector field, superimposing a curve on a vector field.

1. Parametrize each of the following curves.

a. The left half of the circle of radius 3 centered at the origin, starting at (0,-3).

b. The line from the point (1,1) to the point (-1,2).

c. The line from the point (1,0,1) to the point (-1,2,0).

2. Do the lines

$$\vec{r_1} = t\vec{\imath} + (t-1)\vec{\jmath} + (2t-1)\vec{k} \quad (1)$$
$$\vec{r_2} = 2t\vec{\imath} + (t+1)\vec{\jmath} + (3t+1)\vec{k} \quad (2)$$

intersect? If so, where?

3. Find the velocity vector for the curve parametrized by $x = t^2 - t$, $y = \sin(2t)$. Determine when the object is moving parallel to the x-axis and when it is moving parallel to the y-axis.

4. Sketch the vector field $\vec{F} = x\vec{\imath}$.

5. Sketch the vector field $\vec{F} = x\vec{\imath} - y\vec{\jmath}$.

6. Sketch the flow of each of the vector fields above.

7. Parametrize each of the following surfaces:

a. The surface of a sphere of radius 3.

b. The surface of a cone with radius 4 and height 5, centered at the origin.

8. Let $\vec{F}(x,y) = x\vec{\imath} - y\vec{\jmath}$.

a. Use the graph of $\vec{F}$ to determine which of the following parametrized curves could be flow lines for this vector field.

(**i**) $\vec{r}_1(t) = \cos(t)\vec{\imath} - \sin(t)\vec{\jmath}$

(**ii**) $\vec{r}_2(t) = e^t\vec{\imath} - e^t\vec{\jmath}$

(**iii**) $\vec{r}_4(t) = e^t\vec{\imath} + e^{-t}\vec{\jmath}$

(**iv**) $\vec{r}_5(t) = 2e^t\vec{\imath} + e^{-t}\vec{\jmath}$

(**v**) $\vec{r}_1(t) = \cos(t)\vec{\imath} + \sin(t)\vec{\jmath}$

(**vi**) $\vec{r}_5(t) = e^{2t}\vec{\imath} + e^{-2t}\vec{\jmath}$

(**vii**) $\vec{r}_3(t) = e^t\vec{\imath} - e^{-t}\vec{\jmath}$

b. Write down the differential equations associated to the vector field. For each of the the curves that could be flow lines, show that the curve satisfies the differential equations.

9. Let $\vec{F} = y\vec{\imath}$. Determine whether each of the following is positive, negative or zero.

a. $\displaystyle\int_C \vec{F} \cdot d\vec{r}$, where C is the curve $\vec{r}(t) = \cos t\vec{\imath} + \sin t\vec{\jmath}$, $0 \le t \le 2\pi$.

b. $\displaystyle\int_C \vec{F} \cdot d\vec{r}$, where C is the curve $\vec{r}(t) = 2\vec{\imath} + te^{2t}\,\vec{\jmath}$, $0 \le t \le 3$.

10. Let $\vec{F} = x\vec{\imath} + (2x+y)\vec{\jmath} + (3y-z)\vec{k}$, and let C be the circle of radius 2 in the xy-plane traversed clockwise. Compute $\displaystyle\int_C \vec{F} \cdot d\vec{r}$.

11. Let $\vec{F} = xy\vec{\jmath}$. Is $\vec{F}$ path-independent? Why or why not?

12. Let $\vec{F} = x^2\vec{\imath} + e^y\vec{\jmath}$. Is $\vec{F}$ path-independent? Why or why not? Let C be the top half of a circle of radius 4 traversed clockwise. Compute $\displaystyle\int_C \vec{F} \cdot d\vec{r}$.