Math 303, Exam #3 **Name:** ______________________________
Fall 2002

Please write out the full pledge, and sign below.

1. Use the following table of values to find a lower estimate of $\int_R f(x,y)\,dA$, where R is the rectangle $0 \le x \le 1$, $-1 \le y \le 0$.

		x			
		0	.5	1	1.5
	-1	1	5	7	10
y	-.5	2	4	8	11
	0	3	5	9	13
	.5	4	6	11	15

$\frac{1}{4}(1+4+2+4) = \boxed{\frac{11}{4}}$

Compute each of the following integrals.

2. $\int_0^1 \int_{-1}^1 (2x^2+y^2)\,dx\,dy$

$\int_{-1}^1 (2x^2+y^2)\,dx = \frac{2}{3}x^3 + xy^2\Big|_{-1}^1 = \frac{4}{3} + 2y^2$

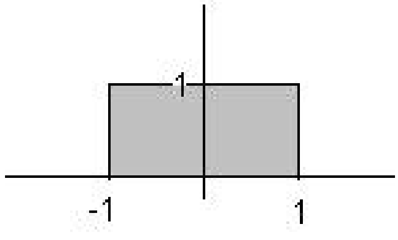
$\int_0^1 \int_{-1}^1 (2x^2+y^2)\,dx\,dy = \int_0^1 \frac{4}{3} + 2y^2\,dy = \frac{4}{3} + \frac{2}{3} = \boxed{2}$

3. $\int_0^1 \int_y^{\sqrt{y}} (2xy - y^2)\,dx\,dy$

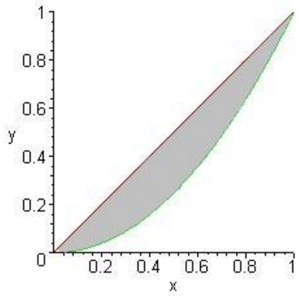
$\int_y^{\sqrt{y}} (2xy-y^2)\,dx = x^2y - xy^2\Big|_y^{\sqrt{y}} = y^2 - y^{7/2}$

$\int_{-1}^1 \int_y^{\sqrt{y}} (2xy-y^2)\,dx\,dy = \int_{-1}^1 y^2 - y^{7/2}\,dy = \boxed{\frac{1}{21}}$

4. Sketch the region of integration for the integrals in each of the last two problems. **2.**

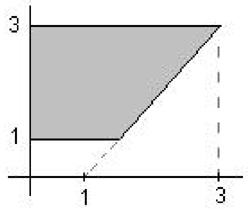


3.



5. For each of the regions, write the integral of $f(x, y) = x^2 - y^2$ over the region as an iterated integral. State which coordinate system you are using.

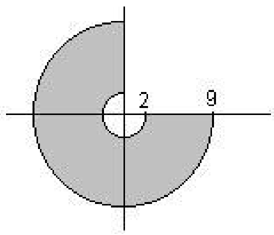
a.



Cartesian

$$\int_1^3 \int_0^{\frac{2}{3}y+1} x^2 - y^2 \, dx \, dy$$

b.

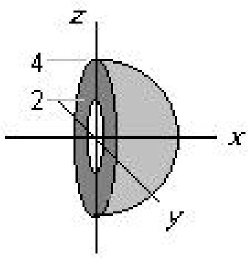


Polar

$$\int_{\frac{\pi}{2}}^{2\pi} \int_2^9 (r^2 \cos^2\theta - r^2 \sin^2\theta) r \, dr \, d\theta$$

6. For each of the following regions, set up a triple integral of $f(x, y, z) = xy + z$ over the region.

a.



Spherical

$$\int_2^4 \int_0^{\pi} \int_{-\pi/2}^{\pi/2} [(\rho \sin\phi \cos\theta)(\rho \sin\phi \sin\theta) + \rho \cos\phi] \rho^2 \, d\rho \, d\phi \, d\theta$$

b. The cylinder of radius 2 and height 4, centered at the origin.

$$\int_{-2}^2 \int_0^{2\pi} \int_0^2 [r^2 \cos\theta \sin\theta + z] r \, dr \, d\theta \, dz$$

7. Parametrize the line from the point (2,4,-1) to the point (0,1,2).

$\boxed{x = 2 - 2t, y = 4 - 3t, z = -1 + 3t}$

8. Find a and b so that the following lines intersect.

$$\begin{aligned} \vec{r}_1(t) &= (t-1)\vec{\imath} + (2t+1)\vec{\jmath} + (3-t)\vec{k} \\ \vec{r}_2(t) &= (t+2)\vec{\imath} + (1-t)\vec{\jmath} + (at+b)\vec{k} \end{aligned}$$

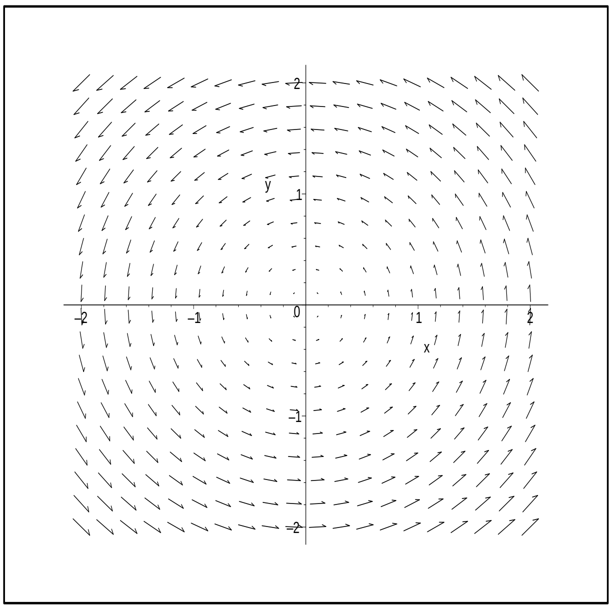
This means finding times t_1 and t_2 so that $\vec{r}_1(t_1) = \vec{r}_2(t_2)$. Then $t_1 - 1 = t_2 + 2$ and $2t_1 + 1 = 1 - t_2$, so $t_1 = t_2 + 3$ and $2(t_2 + 3) + 1 = 1 - t_2$. Then $2t_2 + 6 = -t_2$ so $t_2 = -2$ and $t_1 = 1$. Plugging these times into $\vec{r}_1(t)$ and $\vec{r}_1(t)$ gives $\vec{r}_1(1) = 3\jmath + 2\vec{k}$, and $\vec{r}_2(-2) = 3\jmath + (-2a+b)\vec{k}$. We conclude that $-2a + b = 2$. There are lots of correct choices for a and b, for example $\boxed{a = -1, b = 0}$

9. Sketch the vector field $\vec{F} = y\vec{\imath}$

10. Parametrize the plane through the point $(1, -1, 2)$ that contains the vectors $2\vec{\imath} - \vec{\jmath} + \vec{k}$ and $\vec{\imath} + 3\vec{\jmath} - \vec{k}$

$\boxed{\vec{r}(t) = (1 + 2t + s)\imath + (-1 - t + 3s)\jmath + (2 + t - s)\vec{k}}$

11. The vector field $\vec{F}(x, y) = -y\vec{\imath} + x\vec{\jmath}$ is drawn below.



a. determine which of the following parametrized curves could be flow lines for this vector field.

The flow lines are counterclockwise circles.

(**i**) $\vec{r}_1(t) = 2\cos(t)\vec{\imath} - 2\sin(t)\vec{\jmath}$ Not a flow line (clockwise circle)

(**ii**) $\vec{r}_2(t) = 2\cos(t)\vec{\imath} + 2\sin(t)\vec{\jmath}$

(**iii**) $\vec{r}_3(t) = -2\cos(t)\vec{\imath} + \sin(t)\vec{\jmath}$ Not a flow line (clockwise circle)

(**iv**) $\vec{r}_4(t) = \cos(t)\vec{\imath} - \sin(t)\vec{\jmath}$ Not a flow line (clockwise circle)

(**v**) $\vec{r}_5(t) = \sin(t)\vec{\imath} - \cos(t)\vec{\jmath}$

b. Write down the differential equations associated to the vector field. For each of the the curves that could be flow lines, show that the curve satisfies the differential equations.

Differential equations: $\boxed{\dfrac{dx}{dt} = -y, \dfrac{dy}{dt} = x}$

(ii) $\frac{dx}{dt} = -2\sin t = -y$, $\frac{dy}{dt} = 2\cos t = x$, so (ii) is a flow line for this vector field.
(v) $\frac{dx}{dt} = \cos t = -y$, $\frac{dy}{dt} = \sin t = x$, so (v) is a flow line for this vector field.