Math 303, Final
Fall 2005

Name: ______________________________

Please write out the full pledge and sign below.

Notes
◇ Show all your work to receive full credit.
◇ You may use Maple for any part of this exam. Please email me your Maple files: kbrandlcentenary.edu

1. Let $\vec{v} = \vec{\imath} + 2\vec{\jmath} + \vec{k}$, and $\vec{w} = \vec{\imath} - 2\vec{\jmath} + \vec{k}$.

a. Compute $\vec{v} \cdot \vec{w}$.

b. Compute $\vec{v} \times \vec{w}$.

c. Find the angle θ between $\vec{v}$ and $\vec{w}$.

2. Let $H(x, y) = x \ln(y)$, and let $\vec{v} = \vec{\imath} + \vec{\jmath}$.

a. Find the Taylor polynomial of degree 2 for H near $(1, 1)$.

b. Compute the directional derivative of $H(x, y)$ in the direction of $\vec{v}$, at the point $(1, 1)$.

3. Find the equation for the plane containing the points $(2, 0, 1)$, $(-1, 3, 0)$, and $(4, 4, 0)$.

4. The function $f(x, y)$ is given by

y \ x	10	12	14
1	19	20	21
2	22	24	26
3	27	31	35

a. Approximate $f_x(12, 2)$.

b. Approximate $f_y(12, 2)$.

c. Does $f_{xy}(12, 2)$ appear to be positive, negative or zero? Why?

d. Find an underestimate for $\int_R f \, dA$, where R is the rectangle $10 \leq x \leq 14$, $1 \leq y \leq 3$.

5. Let $g(x, y) = x^2 y$, $x(u, v) = u^2 v - uv^2$, $y(u, v) = \dfrac{u}{v}$. Use the chain rule to compute the partial derivative g_u. Give your answers in terms of u and v.

6. Let $F(x, y) = x^2 y + x - y$. Find all of the critical points. For each critical point decide if it is a local maximum, local minimum, saddle point or none of these.

7. Find the maximum and minimum values of $f(x, y) = 3x^4 + y^4$, subject to the constraint $g(x, y) = x^2 + y^2 \leq 4$.

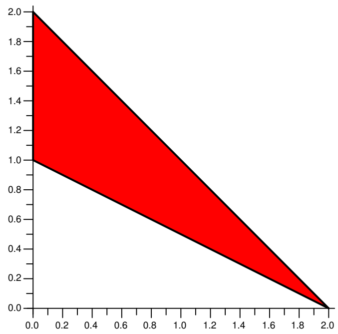
8. Sketch the region of integration for each of the following integrals

a. $\displaystyle\int_{-1}^{2} \int_{x-2}^{2-x} f(x, y)\, dy\, dx$

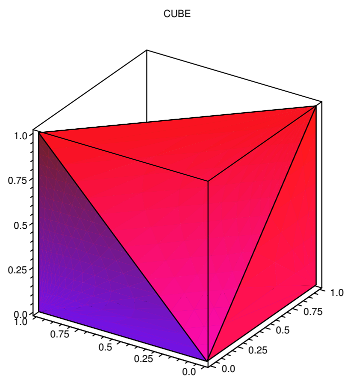
b. $\displaystyle\int_{0}^{\pi} \int_{1}^{3} f(r, \theta)\, dr\, d\theta$

c. $\displaystyle\int_{0}^{4} \int_{0}^{2\pi} \int_{0}^{4-z} f(r, \theta, z)\, dr\, d\theta\, dz$

9. For each of the following regions, set up the integral of a function f over the region. State which coordinate system you are using



a.



b.

10. Let $\vec{F}$ be a three dimensional vector field, f a function of three variables, $\vec{v} = 2\vec{\imath} - 3\vec{\jmath} + 5\vec{k}$ and a, b, and c real numbers. Decide whether each of the following is a number, a point, a function, a vector, or a vector field.

a. $\text{curl}\vec{F}$.

b. $\text{grad}\, f(a, b, c)$.

c. $\vec{F} \cdot \vec{v}$

d. $f_{\vec{v}}$

e. $\text{div}\vec{F}$.

f. $\text{div}\vec{F}(a, b, c)$.

11. Is the following statement valid? Explain.

Let $\vec{F}$ be a vector field such that $\text{div}\vec{F}(x, y, z) = 0$ *for every point* (x, y, z)*. Then for any surface* S, $\displaystyle\int_S \vec{F} \cdot d\vec{A} = 0.$

12. Let $\vec{F}(x,y) = (x+y)\vec{\imath} + y\vec{\jmath}$

a. Write down the differential equations associated to the vector field.

b. Determine which of the following parametrized curves could be flow lines for this vector field. For each of the the curves that could be flow lines, show that the curve satisfies the differential equations.

(i) $\vec{r}_1(t) = e^{-t}\vec{\imath} + e^t\vec{\jmath}$

(ii) $\vec{r}_2(t) = te^t\vec{\imath} + e^t\vec{\jmath}$

(iii) $\vec{r}_4(t) = e^t\vec{\imath} + e^{-t}\vec{\jmath}$

(iv) $\vec{r}_5(t) = 2te^t\vec{\imath} + 2e^t\vec{\jmath}$

13. Let $\vec{F} = 2x\vec{\imath} + 2y\vec{\jmath} + 3xz\vec{k}$.

a. Compute curl$\vec{F}$.

b. Compute div$\vec{F}$.

c. Could $\vec{F}$ be the curl of some other vector field? Explain.

d. Compute $\int_C \vec{F} \cdot d\vec{r}$, where C is the circle of radius 2 in the xy-plane, oriented clockwise with respect to the positive z-axis.

14. Let $\vec{F} = y\vec{\imath} - x\vec{k}$. Determine whether each of the following is positive, negative or zero. Explain, or make a sketch to illustrate your answer.

a. $\displaystyle\int_C \vec{F} \cdot d\vec{r}$, where C is the curve $\vec{r}(t) = \cos t\vec{\imath} + \sin t\vec{\jmath}$, $0 \le t \le \pi$.

b. $\displaystyle\int_C \vec{F} \cdot d\vec{r}$, where C is the curve $\vec{r}(t) = \cos t\vec{\imath} + \sin t\vec{\jmath}$, $0 \le 2\pi$.

c. $\displaystyle\int_S \vec{F} \cdot d\vec{A}$, where S is the rectangle $z = 0$, $-1 \le x \le 1$, $0 \le y \le 1$, oriented upward.

d. $\displaystyle\int_S \vec{F} \cdot d\vec{A}$, where S is the rectangle $y = 0$, $0 \le z \le 1$, $-1 \le x \le 1$, oriented toward the positive y-axis.

e. $\displaystyle\int_S \vec{F} \cdot d\vec{A}$, where S is the rectangle $x = 0$, $0 \le z \le 1$, $0 \le y \le 1$, oriented toward the positive x-axis.

15. Give a careful statement of the Divergence Theorem.

16. Let $\vec{F} = 2xz\vec{\imath} - 3x^2y\vec{\jmath} + 2yz^2\vec{k}$. Let S be a cylinder of radius 4, centered on the z-axis with $0 \le z \le 2$. Use the Divergence Theorem to compute $\displaystyle\int_S \vec{F} \cdot d\vec{A}$.

17. Let $\vec{F} = x\vec{i} + z\vec{j}$.

a. Compute $\int_S \vec{F} \cdot d\vec{A}$, where S is the surface of the rectangular box $0 \le x \le 1$, $0 \le y \le 2$, $0 \le z \le 1$, oriented outward.

b. Compute $\int_S \vec{F} \cdot d\vec{A}$, where S is the part of the surface $f(x,y) = x^2$ over the rectangle $0 \le x \le 3$, $0 \le y \le 2$.

c. Compute $\int_S \vec{F} \cdot d\vec{A}$, where S is the cylinder of radius 4 centered on the z-axis with $0 \le z \le 3$.

18. Let $f(x,y) = 4\sqrt{y^2+4} + x\cos(xy)$. Use the graph of $f(x,y)$ to approximate the maximum and minimum values of $f(x,y)$, subject to the constraint $g(x,y) = x^2 + y^2 \le 4$.

19. Let $f(x,y) = y \sin x \cos y$, and $\vec{v} = \vec{i} + \vec{j}$.

a. Use the graph of $f(x,y)$ to locate a point (a,b) such that $f(a,b)$ is defined and $\operatorname{grad} f(a,b) = \vec{0}$.

b. Use the graph of $f(x,y)$ to determine whether $f_{\vec{v}}(2,2)$ is positive, negative or nearly zero.

c. Approximate $\operatorname{grad} f(2,2)$.