Math 203, Exam #3 Review **Fall 2009**

Your final exam will be cumulative, and will cover §1.1-1.5, 1.7-1.10, 2.1-2.5, 2.7,2.8, 3.1,3.2, 4.1-4.6, 4.9, 5.1-5.3, 6.1-6.4, 7.1. In addition to the problems below, you should look over your homework - **including the true/false questions**, and the review sheets from each of the 3 previous exams. **Be prepared to justify your answers**. Topics we've studied include: Systems of Linear Equations using Matrices, Echelon Form and Reduced Echelon Form, Vector Equations and Matrix Equations, Applications of Linear Algebra Linear Independence, Linear Transformations, Matrix Algebra, the Inverse of a Matrix, Characteristics of Invertible Matrices: Statements that are equivalent to "A is an invertible matrix," Partitioned Matrices LU Factorization, Homogeneous Coordinates, Subspaces of $\mathbb{R}^n$, Determinants, Vector Spaces, Subspaces, Spanning set, Bases, Null Space, Column Space, Row Space, Change of coordinates, Dimension, Rank. Markov Chains, Eigenvectors and Eigenvalues, The Characteristic Equation, Diagonalization, Inner Product, Length, Orthogonal Sets, Orthogonal Projection. Orthonormal sets, The Gram-Schmidt Process, Symmetric Matrices

Here are some problems from §4.9 on. Be sure you know how to do these problems without Maple.

1. A particular strain of flu is going around campus. There are students who have never had the flu, students who have the flu now, and students who are recovered from the flu and now immune. Each week, 5% of the students who never had the flu will get the flu and 20% of the student with flu will recover and become immune.

a. Set up the stochastic matrix for this situation.

b. If we start with 80% of the students never having had the flu, and 20% having the flu now, what will the situation be after 10 weeks assuming the trend continues?

c. Is there a steady state for this situation? If so, find it.

2. Each month, 10% of the employed people lose their job, and half of the unemployed people get a job.

a. Set up the stochastic matrix for this situation.

b. Starting with 7% unemployment, what will the situation be in one year, assuming this trend continues?

c. Is there a steady state for this situation? If so, find it.

3. Is $\lambda = 10$ an eigenvalue for $M = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 2 \\ 5 & 3 & 5 \end{bmatrix}$? If so describe the eigenvectors with eigenvalue 10.

4. Is $\begin{bmatrix} 7 \\ 6 \\ 5 \end{bmatrix}$ an eigenvector for $\begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 2 \\ 2 & 1 & 1 \end{bmatrix}$? If so, find the corresponding eigenvalue.

5. Find the eigenvalues for $\begin{bmatrix} 2 & 0 & 0 \\ 3 & 1 & 0 \\ 3 & 4 & -1 \end{bmatrix}$. For each eigenvalue, find an eigenvector.

6. Find the characteristic equation for the matrix $A = \begin{bmatrix} 2 & 1 & 3 \\ -3 & 2 & 2 \\ 1 & 5 & 1 \end{bmatrix}$.

7. Find the characteristic equation for the matrix $\begin{bmatrix} 2 & 2 \\ 3 & 4 \end{bmatrix}$, and use it to find the eigenvalues of this matrix.

8. Let $P = \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$, and $D = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$, and let $A = PDP^{-1}$. Compute A^8.

9. Let $\vec{v} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$, and $\vec{w} = \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}$.

a. Compute $\vec{v} \cdot \vec{w}$, and $||\vec{v}||$.

b. Find a unit vector in the direction of $\vec{v}$.

c. Find the distance between $\vec{v}$ and $\vec{w}$.

d. Find a nonzero vector that is orthogonal to $\vec{v}$.

10. Decide which of the following sets are orthogonal.

a. $\left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix} \right\}$

b. $\left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

11. Give an example of an orthonormal basis for $\mathbb{R}^3$, other than the standard basis.

12. Let $\vec{u}_1 = \begin{bmatrix} 1 \\ -4 \\ -1 \\ 0 \end{bmatrix}, \vec{u}_2 = \begin{bmatrix} 5 \\ 1 \\ 1 \\ 3 \end{bmatrix}, \vec{u}_3 = \begin{bmatrix} 0 \\ 1 \\ -4 \\ 1 \end{bmatrix}$.

a. Show that $\{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$ is a linearly independent set. There is a way to do this with minimal computation.

b. Let W be the span of $\{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$. Find an orthonormal basis for W.

c. Let $\vec{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$. Find the orthogonal projection of $\vec{x}$ onto W.

d. Find the distance from $\vec{x}$ to W.

e. Find the dimension of $W^{\perp}$

f. Find a basis for $W^{\perp}$.

13. Use the Gram-Schmidt process to find an orthonormal basis for the span of the vectors:

$$\begin{bmatrix} 1 \\ 2 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

14. Find the QR Factorization of the matrix $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 0 \\ -1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

15. Orthodgonally diagonalize the matrix $\begin{bmatrix} 1 & 1 & -2 \\ 1 & 1 & 2 \\ -2 & 2 & 2 \end{bmatrix}$.