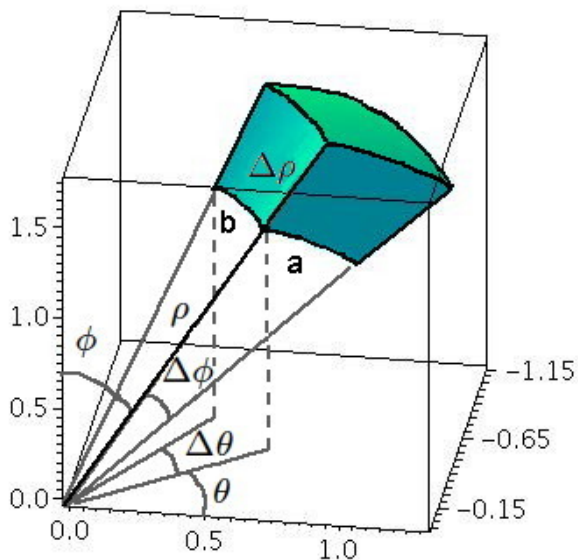


In order to integrate over a 3d region described in spherical coordinates, we need to find the volume element, shown below.

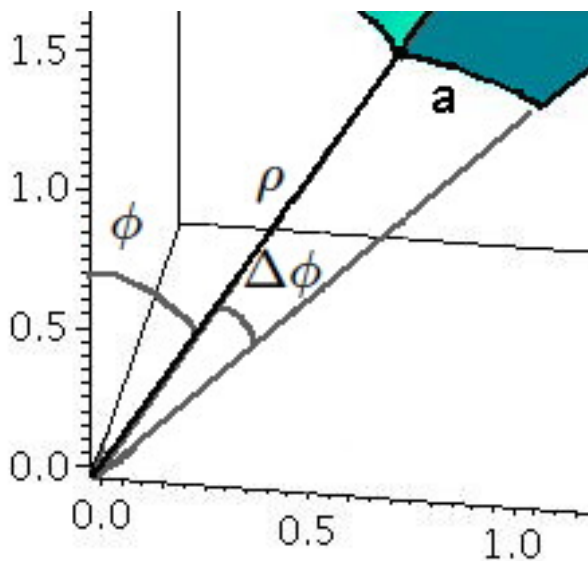


While this is not a box, its volume can be approximated by taking the product of its edges, labeled a , b and $\Delta\rho$:

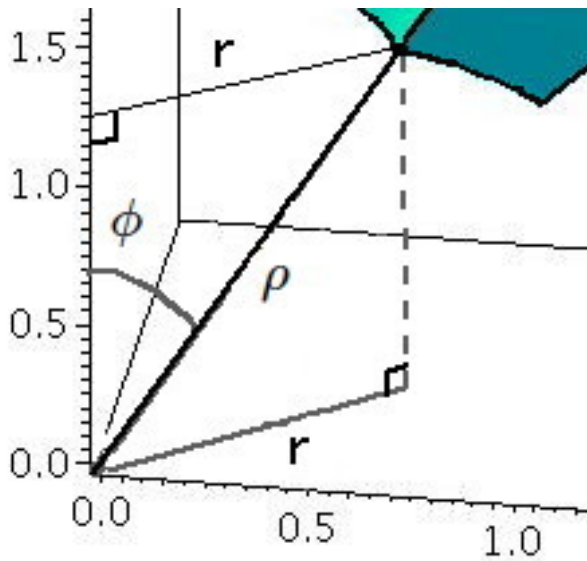
$$\Delta V \approx ab\Delta\rho$$

So we will know the volume element if we can compute a and b .

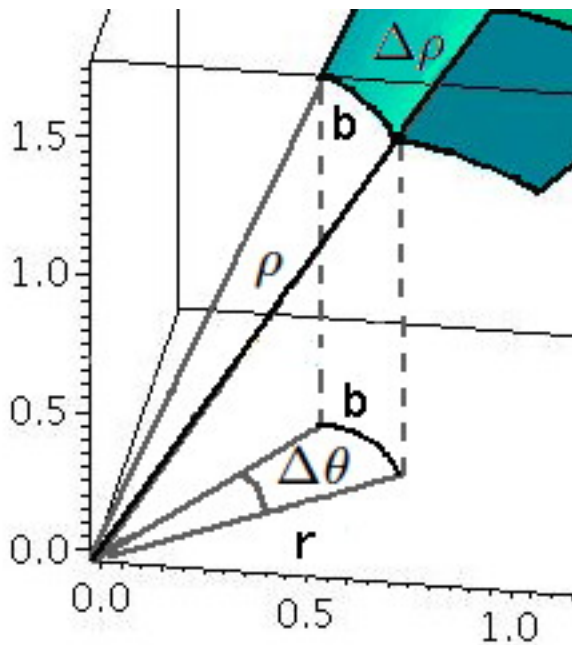
1. Use the arclength formula, $s = r\theta$ to compute a .



2. Using the image below, find a formula for r in terms of ρ and ϕ .



3. Now use the image below and the arclength formula again to find b .



4. Putting this all together, $\Delta V \approx ab\Delta\rho =$ _____
5. Use the volume element to set up the integral of a function f over the region described by $-1 \leq \rho \leq 1, 0 \leq \theta \leq \frac{\pi}{2}, \frac{\pi}{3} \leq \phi \leq \frac{2\pi}{3}$.