Math 303, Worksheet #11 **Name:** ____________________________
Fall 2008

Recall that two parametrized curves *intersect* there is a point (a, b) that lies on both curves. There is a *collision* if the same point is on both curves with the same value of t.

To check to see if there is a collision, we set the functions for x equal to each other, functions for y equal to each other and functions for z equal to each other.

1. Check to see if the following pairs of lines collide:

a. $l_1: \begin{array}{rcl} x &=& 1-t \\ y &=& 2-2t \\ z &=& 4+t \end{array}$ $\qquad l_2: \begin{array}{rcl} x &=& -2+t \\ y &=& -4-3t \\ z &=& 7+2t \end{array}$

b. $l_1: \begin{array}{rcl} x &=& 3-2t \\ y &=& 5+t \\ z &=& -4+2t \end{array}$ $\qquad l_2: \begin{array}{rcl} x &=& -1+2t \\ y &=& 3+3t \\ z &=& -1-t \end{array}$

c. $l_1: \begin{array}{rcl} x &=& 3-2t \\ y &=& 5+t \\ z &=& -4+2t \end{array}$ $\qquad l_2: \begin{array}{rcl} x &=& 7+6t \\ y &=& 1-3t \\ z &=& 5-6t \end{array}$

To check if two parametrized lines to intersect, plug t_1 into the first and plug t_2 into the second, and set the functions for x equal to each other, functions for y equal to each other and functions for z equal to each other. This will give you three equations in t_1 and t_2. If there is a solution to the system of equations, then there is an intersection, otherwise there is no intersection. Check to see if the pairs of lines above intersect. You will probably need to use the back of this page for your computations.

For a parametrized curve, the velocity vector is a vector $\vec{v}$ that points in the direction of travel, and whose magnitude $||\vec{v}||$ is the speed. As you might expect, the velocity vector is the derivative of the position vector. As you might also expect, the acceleration vector, $\vec{a}$ is a vector that describes the direction and magnitude of the acceleration, and it is found by taking the derivative of the velocity vector.

For a parametrized curve, $x = f(t)$, $y = g(t)$, $z = h(t)$, the position, velocity and acceleration vectors are given by:

$$\begin{aligned} \vec{r}(t) &= f(t)\vec{\imath} + g(t)\vec{\jmath} + h(t)\vec{k} \\ \vec{v}(t) = \vec{r}\,'(t) &= f'(t)\vec{\imath} + g'(t)\vec{\jmath} + h'(t)\vec{k} \\ \vec{a}(t) = \vec{v}\,(t) = \vec{r}\,''(t) &= f''(t)\vec{\imath} + g''(t)\vec{\jmath} + h''(t)\vec{k} \end{aligned}$$

2. Find the velocity and acceleration vectors for each of the following curves.

a. $x = t$, $y = t^3 - t$

b. $x = 2 + 3t$, $y = 4 + t$, $z = 1 - t$

c. $x = 3\cos t$, $y = 4\sin t$

3. Find the velocity and the speed for each of the following curves. Does the object ever stop? If so where?

a. $x = 3t^2$, $y = t^3 + 1$

b. $x = (t-1)^2$, $y = 2$, $z = 2t^3 - 3t^2$