Math 303, Worksheet #16 **Name:** ____________________________
Fall 2008

Yesterday we looked at the Fundamental Theorem of Line Integrals:

If the vector field $\vec{F}$ is the gradient of a function f, and C is a parametrized curve that starts at the point P and ends at the point Q, then

$$\int_C \vec{F} \cdot d\vec{r} = f(Q) - f(P).$$

It is important to note that this means that if $\vec{F} = \text{grad}\, f$, then $\int_C \vec{F} \cdot d\vec{r}$ depends only on the endpoints of C, and not on the path itself. We call a vector field with this property *path independent.*

BTW: *Fundamental Theorem* - do you think this will be on the exam? :)

1. Let

$$f(x, y) = x + xy + y^2,$$

and C the parametrized curve given by

$$\vec{r}(t) = 2\cos(t)\vec{\imath} + 2\sin(t)\vec{\jmath}, \quad 0 \leq t \leq \pi.$$

Use the Fundamental Theorem to compute $\displaystyle\int_C \text{grad}\, f \cdot d\vec{r}$

2. The Fundamental Theorem can be used for 3d line integrals too. Let

$$f(x, y, z) = x^2 - yz$$

and C the parametrized curve given by

$$\vec{r}(t) = 2\cos(t)\vec{\imath} + 2\sin(t)\vec{\jmath} + t^2\vec{k}, \quad 0 \leq t \leq \pi.$$

Use the Fundamental Theorem to compute $\displaystyle\int_C \text{grad}\, f \cdot d\vec{r}$

3. Suppose $\vec{F} = \text{grad}\, f$ for some function f, and C is the curve parametrized by $\vec{r}(t) = \cos(t)\vec{\imath} + \sin(t)\vec{\jmath}$, $0 \le t \le 2\pi$. What is $\int_C \vec{F} \cdot d\vec{r}$? What is it about C that makes this work?

4. In order to use the Fundamental Theorem to compute $\int_C \vec{F} \cdot d\vec{r}$, we will need to be able to find the function f. For each of the following vector fields, find a function f so that $\vec{F} = \text{grad}\, f$. This will involve guessing and checking.

a. $\vec{F}(x, y) = 2x\vec{\imath} + 2y\vec{\jmath}$

b. $\vec{F}(x, y) = y\vec{\imath} + x\vec{\jmath}$

c. $\vec{F}(x, y) = y\vec{\imath} + (x + 2)\vec{\jmath}$

d. $\vec{F}(x, y, z) = yz\vec{\imath} + xz\vec{\jmath} + xy\vec{k}$

Not all vector fields are the gradient of a function f.

5. Let $\vec{F} = x\vec{\jmath}$.

a. For each of the curves below, decide whether $\int_C \vec{F} \cdot d\vec{r}$ is positive, negative or zero.

(**i**) C is the straight line from the point $(1, 0)$ to the point $(-1, 0)$.

(**ii**) C is the top half of the circle of radius 1 centered at the origin, from the point $(1, 0)$ to the point $(-1, 0)$.

b. What can you conclude about $\vec{F}$?

6. Let $\vec{F} = -y\vec{\imath} + x\vec{\jmath}$.

a. Find curves C_1 and C_2 that start at the same point and end at the same point, but

$$\int_{C_1} \vec{F} \cdot d\vec{r} \neq \int_{C_2} \vec{F} \cdot d\vec{r}.$$

You should be able to do this without actually computing the integrals.

b. What can you conclude about $\vec{F}$?