Math 303, Worksheet #17 **Name:** ______________________________
Fall 2008

Vector Fields

1. For each of these vector fields, compute $\text{curl}(\vec{F_i})$, and determine whether the vector field is path-independent. For each of the path-independent vector fields, find a potential function f. [Recall that a potential function is a function f so that $\vec{F} = \text{grad}\, f$].

.	$\text{curl}(\vec{F})$	path independent?	potential, f
$\vec{F_1} = 2xy\vec{\imath} + x^2\vec{\jmath}$			
$\vec{F_2} = xy^2\vec{\jmath}$			
$\vec{F_3} = \vec{\imath} + \vec{\jmath}$			
$\vec{F_4} = 2e^x\vec{\imath}$			
$\vec{F_5} = \sqrt{1-x^2}\vec{\jmath}$			
$\vec{F_6} = 2y\vec{\imath} + (2x-2y)\vec{\jmath}$			
$\vec{F_7} = x\vec{\imath} + xy^2\vec{\jmath}$			

2. Suppose C is a closed curve. Compute $\int_C \vec{F_i} \cdot d\vec{r}$ for each of the path-independent vector fields above.

We now have two methods for computing a line integral for a closed curve.

(I.) The old way:

$$\int_C \vec{F} \cdot d\vec{r} = \int_a^b \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)\, dt,$$

Where $\vec{r}(t)$ is a parametrization for C with $a \le t \le b$.

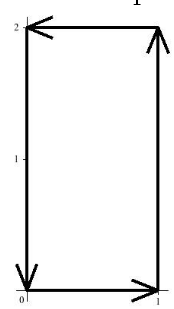
(II.) The new way uses the formula from yesterday:

$$\int_C \vec{F} \cdot d\vec{r} = \int_R \text{curl}(\vec{F})\, dx\, dy,$$

Where R is the region in the xy-plane enclosed by C, and C travels around R in a counter-clockwise direction.

3. Let C be the circle parametrized by $\vec{r}(t) = \cos(t)\vec{\imath} + \sin(t)\vec{\jmath}$, $0 \leq t \leq 2\pi$. Use each of the two methods to compute $\int_C \vec{F_i} \cdot d\vec{r}$ for each of the vector fields on page one that are not path-independent.

4. Let D be the parametrized curve that starts and ends at $(0, 0)$ drawn below.



a. Compute $\int_D \vec{F_i} \cdot d\vec{r}$ for each of the vector fields on page one. You may use whichever method you like.

b. How should your answer change if we travel the same path in the clockwise direction?