Math 303, Worksheet #18 **Name: Solutions**
Fall 2008

Yesterday we looked at the formula for a flux integral when the surface is a cylinder (oriented outward):

$$\int_S \vec{F} \cdot d\vec{A} = \int_0^{2\pi} \int_a^b \vec{F}(r, \theta, z) \cdot (\cos\theta \vec{\imath} + \sin\theta \vec{\jmath}) R \, dz \, d\theta$$

Where a and b are the z-values that determine the height of the bottom and top of the cylinder. Observe that $\vec{F}$ must be converted to cylindrical coordinates. You can see how to use this formula to compute the flux across a section of a cylinder by adjusting the limits for θ.

1. Compute $\int_{S_1} \vec{F_1} \cdot d\vec{A}$ where S_1 is the cylinder of radius 9 and height 4, with base in the xy-plane, centered on the z-axis, $\vec{F_1} = zy\vec{\imath} + zx\vec{\jmath} + z^2\vec{k}$.

$\vec{F_1}(r, \theta, z) = 9z\sin\theta\vec{\imath} + 9z\cos\theta\vec{\jmath} + z^2\vec{k}$

$\int_{S_1} \vec{F_1} \cdot d\vec{A} = \int_0^{2\pi} \int_0^4 162\, z \cos\theta \sin\theta \, dz \, d\theta = \boxed{0}$

2. Quick! Compute $\int_{S_1} \vec{F_2} \cdot d\vec{A}$ where $\vec{F_2} = zy\vec{\imath} + zx\vec{\jmath} + z^2xe^z\sin(xy)\vec{k}$, and S_1 is the same cylinder.

$\boxed{0}$ For the sides of a cylinder, the $\vec{k}$ component of the vector field doesn't matter.

3. Compute $\int_{S_2} \vec{F_1} \cdot d\vec{A}$ where S_2 is the $\frac{1}{4}$-cylinder of S_1 in the positive xyz-octant.

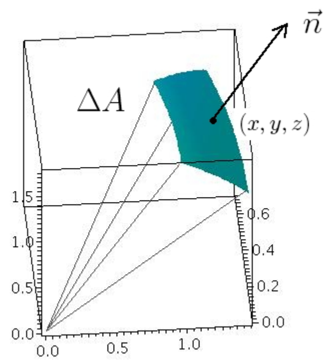
As with line integrals, you can graph the surface and the vector field to see if you answer seems reasonable. The Maple command for plotting a surface in cylindrical coordinates is:

plot3d("function for r", t = a..b , z = c..d, coords = cylindrical, color = black)

I've included color=black to make the vectorfield stand out when you plot the vector field with the surface. I'm also using t for θ.

4. Plot the surface S together with the vectorfield $\vec{F_1}$ to see if your answers are reasonable.

In order to compute flux integrals over spheres, we need to find $\Delta\vec{A}$ for a sphere. This is very much like finding the volume element for a sphere (which you did on a previous worksheet). Start with a Sphere of radius R centered a the origin, and carve it up into pieces ΔA, each of which looks like the image below.



In spherical coordinates, the area of ΔA is approximated by:

$\Delta A \approx R^2 \sin\phi\, \Delta\theta\Delta\phi$

$\Delta\vec{A}$ is the vector that is perpendicular to ΔA with magnitude equal to the area of ΔA, so $\Delta\vec{A} = \Delta A\,\vec{n}$, where $\vec{n}$ is a unit vector perpendicular to the the surface ΔA.

5. Say $\vec{n}$ starts at a point (x, y, z) Find the formula for $\vec{n}$ in terms of x, y and z.

6. Convert your formula for $\vec{n}$ into spherical coordinates.

7. Find $\Delta\vec{A} = \Delta A\,\vec{n}$

8. You are now ready to write down the formula for flux integrals in spherical coordinates:

$$\int_S \vec{F} \cdot \Delta\vec{A} =$$

Check your formula with me before computing the following integrals.

9. $\displaystyle\int_{S_3} \vec{F_1} \cdot d\vec{A}$ where S_3 is the sphere of radius 9 centered at the origin.

10. $\displaystyle\int_{S_4} \vec{F_1} \cdot d\vec{A}$ where S_4 is the $\frac{1}{8}$-sphere of S_3 in the positive xyz-quadrant.

11. Again, you can check your answers by graphing this surface with the vector field. The Maple command is:

plot3d("function for rho", t = a..b, f = c..d, coords = spherical, color = black)
I'm using t for θ and f for ϕ.