Math 303, Worksheet #19 **Name:** _____
Fall 2008

Yesterday we saw the formula for computing the flux through a surface defined by $z = f(x, y)$ with x, y in a region R:

$$\int_S \vec{F} \cdot d\vec{A} = \int_R \vec{F}(x, y, f(x, y)) \cdot (-f_x\vec{\imath} - f_y\vec{\jmath} + \vec{k})\, dx\, dy.$$

Then we saw the formula for computing the flux through a surface defined by a parametrization $\vec{r}(s, t)$:

$$\int_S \vec{F} \cdot d\vec{A} = \int_R \vec{F}(\vec{r}(s, t)) \cdot \left(\frac{\partial \vec{r}}{\partial s} \times \frac{\partial \vec{r}}{\partial t}\right) ds\, dt.$$

1. Let S_1 be the surface defined by $z = x^2 - y^2$ with $-2 \le x \le 2$ and $-2 \le y \le 2$, and let $\vec{F} = xy\vec{\imath} - yz\vec{\jmath} + zx\vec{k}$.

a. Compute $\displaystyle\int_S \vec{F} \cdot d\vec{A}$.

b. Repeat part **a** using the surface S_2, defined by the same function over the disk with radius 1 centered at the origin.

2. Let S_3 be the surface parametrized by $\vec{r}(s, t) = s^3t\vec{\imath} + s^2t^2\vec{\jmath} + st^3\vec{k}$

a. Compute $\displaystyle\int_{S_3} \vec{G} \cdot d\vec{A}$ where $\vec{G} = x\vec{\imath} + y\vec{\jmath} + z\vec{k}$.

b. Repeat part **a** with the vector field $\vec{H} = x\vec{\imath} - y\vec{\jmath} + z\vec{k}$

3. On the last worksheet we constructed the formula for flux through a sphere of radius R:

$$\int_S \vec{F} \cdot d\vec{A} = \int_T \vec{F}(R, \phi, \theta) \cdot (\sin\phi\cos\theta\vec{\imath} + \sin\phi\sin\theta\vec{\jmath} + \cos\phi\vec{k}) R^2 \sin\phi \, d\phi \, d\theta$$

a. Compute the flux across a sphere S_R of radius R for the vector field $\vec{G} = x\vec{\imath} + y\vec{\jmath} + z\vec{k}$. This should simplify nicely.

b. Let V be the volume of S_R. Compute $\dfrac{\int_{S_R} \vec{G} \cdot d\vec{A}}{V}$. There should be a formula for the volume of a sphere in the cover of your text.

4. For a vector field $\vec{F} = F_1\vec{\imath} + F_2\vec{\jmath} + F_3\vec{k}$, the *divergence* of $\vec{F}$ is defined by

$$\text{div}\vec{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$$

Compute the divergence of each of the following vector fields.

a. $\vec{G} = x\vec{\imath} + y\vec{\jmath} + z\vec{k}$

b. $\vec{F} = xy\vec{\imath} - yz\vec{\jmath} + zx\vec{k}$