Math 303, Worksheet #20 **Name:** ______________________________
Fall 2008

Yesterday we looked at the curl of a 3 dimensional vector field, $\vec{F} = F_1\vec{\imath} + F_2\vec{\jmath} + F_3\vec{k}$:

$$\operatorname{curl}\vec{F} = \left(\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}\right)\vec{\imath} + \left(\frac{\partial F_1}{\partial z} - \frac{\partial F_3}{\partial x}\right)\vec{\jmath} + \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right)\vec{k}$$

1. Compute the curl of each of the following vector fields:

a. $\vec{F} = y\vec{\imath} + z\vec{\jmath} + x\vec{k}$

b. $\vec{G} = xy\vec{\imath} + yz\vec{\jmath} + xz\vec{k}$

c. $\vec{H} = (xy + z)\vec{\imath} + (yz + x)\vec{\jmath} + (xz + y)\vec{k}$

d. $\vec{K} = (2xyz)\vec{\imath} + x^2z\vec{\jmath} + 3xy\vec{k}$

Here's another Theorem to add to our "List of important Theorems."

Theorem 0.1. [Stokes] *Let S be a surface with boundary C. Assume C is oriented so that the surface S is on the left as you travel along C. Let $\vec{F}$ be a differentiable vector field on S and C. Then*

$$\int_C \vec{F} \cdot d\vec{r} = \int_S \operatorname{curl} \vec{F} \cdot d\vec{A}.$$

2. Before we apply this Theorem, we should understand both sides of the equation.

a. What type of integral is $\int_C \vec{F} \cdot d\vec{r}$?

b. What type of integral is $\int_S \operatorname{curl} \vec{F} \cdot d\vec{A}$?

3. Let C be the circle with radius 2 in the xy-plane, and let $\vec{F} = y\vec{\imath} + z\vec{\jmath} + x\vec{k}$ (as on page 1).

a. In order to use Stokes' Theorem to compute $\int_C \vec{F} \cdot d\vec{r}$, we need to first describe a surface S whose boundary is C. Describe the surface S, first in words, then as either a parametrized surface or as the graph of a function $z = f(x, y)$ over a region in the $xy-$plane.

b. Compute $\int_S \operatorname{curl} \vec{F} \cdot d\vec{A}$, using one of the formulas from section 19.2 or 19.3.

c. Now compute this integral using the original formula (from section 18.2)

4. Let B be the ellipse in the plane $z = y$ described by $\vec{r}(t) = \cos(t)\vec{\imath} + \sin(t)\vec{\jmath} + \sin(t)\vec{k}$.

a. Describe a surface S whose boundary is C, first in words, then as the graph of a function $z = f(x, y)$ over a region in the xy-plane. It may help to graph $\vec{r}(t)$ using Maple.

b. Compute $\displaystyle\int_S \operatorname{curl} \vec{F} \cdot d\vec{A}$, using one of the formulas from section 19.2.

c. Now, compute $\int_B \vec{F} \cdot d\vec{r}$ using the original formula.