Math 303, Worksheet #21 **Name:** ______________________________

Fall 2008

Let:
$\vec{F}_1(x,y,z) = x^2\vec{\imath} + y^2\vec{\jmath} + z^2\vec{k}$
$\vec{F}_2(x,y,z) = x\vec{\imath} + y\vec{\jmath} - z\vec{k}$
$\vec{F}_3(x,y,z) = yz\vec{\imath} + x^2\vec{\jmath} + y\vec{k}$

1. Compute:

a. $\text{div}\vec{F}_1$

b. $\text{div}\vec{F}_2$

c. $\text{div}\vec{F}_3$

2. Compute:

a. $\text{curl}\vec{F}_1$

b. $\text{curl}\vec{F}_2$

c. $\text{curl}\vec{F}_3$

3. S be the surface parametrized by $\vec{r}(t) = 3s^2\vec{\imath} + (4s + t^2)\vec{\jmath} + 2t\vec{k}$, $0 \le s \le 1$, $0 \le t \le 1$, oriented in the positive x direction.

a. Compute $\displaystyle\int_S \vec{F}_1 \cdot d\vec{A}$

4. Suppose $\vec{F}$ is a continuous vector field and $\text{curl}\vec{F} = \vec{0}$, and that C is any closed curve. What is $\displaystyle\int_C \vec{F} \cdot d\vec{r}$? Explain.

5. Suppose $\vec{F}$ is a continuous vector field and $\text{div}\vec{F} = 0$, and that S is any closed surface. What is $\displaystyle\int_S \vec{F} \cdot d\vec{A}$? Explain.

6. Let S be the surface of a sphere of radius 4 centered at the origin, and let C be the ellipse given by $\vec{r}(t) = \cos(t)\vec{\imath} + \sin(t)\vec{\jmath} + \sin(t)\vec{k}$ Compute each of the following:

7. $\int_C \vec{F}_1 \cdot d\vec{r}$

8. $\int_C \vec{F}_2 \cdot d\vec{r}$

9. $\int_C \vec{F}_3 \cdot d\vec{r}$

10. $\int_S \vec{F}_1 \cdot d\vec{A}$

11. $\int_S \vec{F}_2 \cdot d\vec{A}$

12. $\int_S \vec{F}_3 \cdot d\vec{A}$

13. There was more than one way to solve each of the problems above. For each one, set up the integral for the method you didn't use.