Math 303, Worksheet #5 **Name:** ______________________

Fall 2008

Let $g(x,y) = x^2 - 2xy - y^2$, and $v = \vec{\imath} + \vec{\jmath}$.

1. Compute $g_x(x,y)$ and $g_y(x,y)$.

2. Given a function $f(x,y)$ and a point (a,b), the gradient of f at (a,b) is

$$\operatorname{grad} f(1,1) = f_x(a,b)\vec{\imath} + f_y(a,b)\vec{\jmath}$$

Observe that this is a two dimensional *vector*.

Compute the gradient $\operatorname{grad} g(a,b)$ at each of the following points:

a. $(a,b) = (1,1)$

b. $(a,b) = (-1,1)$

c. $(a,b) = (1,-2)$

3. Compute $\operatorname{grad} p(a,b)$ for the function $p(x,y) = 3x + 2y + 3$ for each of the following points:

a. $(a,b) = (1,1)$

b. $(a,b) = (-1,1)$

Given a function $f(x,y)$, a point (a,b) and a vector $\vec{u}$, the directional derivative $f_{\vec{u}}(a,b)$ is the slope of the function $f(x,y)$ at the point (a,b) in the direction of vector $\vec{u}$. Since slope is rise over run, we should be able to approximate this by computing $\dfrac{\Delta f}{\Delta(x,y)}$. Here $\Delta(x,y)$ should mean change in position in the (x,y) plane as we travel along $\vec{u}$, and Δf should mean the resulting change in $f(x,y)$.

4. Let $f(x,y) = 3x + 2y + 3$ and $\vec{u} = \vec{\imath} + \vec{\jmath}$. We want to compute $f_{\vec{u}}(1,2)$, but I'll break it down into steps:

a. Starting at the point $P = (1,2)$, if we travel along $\vec{u}$, which point in the $xy-$plane is our ending point? Call this point Q.

b. Starting at the point $P = (1,2)$, if we travel along $\vec{u}$ to point Q, how far have we travelled in the $xy-$plane? Call this $\Delta(x,y)$.

c. Compute the value of the function $f(x, y)$ at each of the points P and Q

d. Compute Δf as we travel from point P to point Q.

e. Now compute $\dfrac{\Delta f}{\Delta(x, y)}$.

f. If instead of traveling the whole length of $\vec{u}$, we only travelled halfway along $\vec{u}$, what would we get for $\dfrac{\Delta f}{\Delta(x, y)}$? What is it about $f(x, y)$ that makes this work?

g. Repeat this problem using the same $f(x, y)$, the same $P = (1, 2)$, but using the vector $\vec{v} = 3\vec{\imath} + 2\vec{\jmath}$.

5. With the function $g(x, y) = 2x^3y$ compute each of the following:

a. $g(1, 2)$ and $g(1.2, 2.4)$;

b. $g(1.2, 2.4) - g(1, 2)$; (call this Δg)

c. the distance from the point (1,2) to the point (1.2,2.4); (call this $\Delta(x, y)$).

d. $\dfrac{\Delta g}{\Delta(x, y)}$.

e. Which directional derivative does $\dfrac{\Delta g}{\Delta(x, y)}$ approximate? Which computation should we make to get a better approximation? (You don't actually have to perform the computation).