Math 303, Worksheet #6 **Name:** _____
Fall 2008

Let $g(x, y) = x^2 - 2xy - y^2$, and $v = \vec{\imath} + \vec{\jmath}$.

1. Compute $g_x(x, y)$ and $g_y(x, y)$.

2. Compute grad $g(1, 2)$.

3. Compute $g_{\vec{v}}(1, 2)$. Note: $\|\vec{v}\| \neq 1$.

Problems 4 through 6 require you to print your Maple graphs. Please avoid printing more pages than necessary. Also, include some identification before printing so you can distinguish your printouts from the others.

4. Use Maple to graph the function $g(x, y)$. Mark the point on the graph where $x = 1$, $y = 2$ on a printout of this graph.

5. Now change your graph to contour style, and change the perspective to get a contour diagram of $g(x, y)$. On a printout of this graph, starting at the point (1,2), draw an arrow in the direction of the gradient vector grad $g(1, 2)$.

6. Let $f(x, y) = x^3y^3$

a. For each of the following points (a, b), compute grad $f(a, b)$: $(2, 2), (2, -2), (-2, 2), (-2, -2)$.

b. For each of the following $\vec{v}$, compute $f_{\vec{v}}(2, 2)$

(**i**) $\vec{v} = \vec{\imath} + \vec{\jmath}$

(**ii**) $\vec{v} = \vec{\imath} - \vec{\jmath}$

c. Graph the function $f(x, y)$ in contour style and change the perspective to get a contour diagram of $f(x, y)$, and print a copy of this graph. For each of the points (a, b) listed in part **a**, mark the direction of the gradient vector on the graph of $f(x, y)$ starting at the point (a, b). While you're at it, check that each of your answers in parts **a** and **b** make sense.

Why stop there? These ideas generalize to functions of three variables: For a function of three variables, $f(x, y, z)$:

$$\text{grad } f(a, b, c) = f_x(a, b, c)\vec{\imath} + f_y(a, b, c)\vec{\jmath} + f_z(a, b, c)\vec{k},$$

and the directional derivative of $f(x, y, z)$ at (a, b, c) in the direction of the unit vector $\vec{u} = u_1\vec{\imath} + u_2\vec{\jmath} + u_3\vec{k}$ is

$$f_{\vec{u}}(a, b, c) = \text{grad } f(a, b, c) \cdot \vec{u} = f_x(a, b, c)u_1 + f_y(a, b, c)u_2 + f_z(a, b, c)u_3$$

7. Let $f(x, y, z) = 3x^2y^3 + 4z^2$ Compute each of the following:

a. grad $f(1, 2, -1)$

b. $f_{\vec{u}}(1, 2, -1)$, where $\vec{u} = 2\vec{\imath} + \vec{\jmath} - \vec{k}$.