Math 303, Worksheet #6 **Name: Solutions**
Fall 2008

Let $g(x,y) = x^2 - 2xy - y^2$, and $v = \vec{\imath} + \vec{\jmath}$.

1. Compute $g_x(x,y)$ and $g_y(x,y)$.

$\boxed{g_x(x,y) = 2x - 2y}$ and $\boxed{g_y(x,y) = -2x - 2y}$

2. Compute $\operatorname{grad} g(1,2)$.

$g_x(1,2) = 2 - 4 = -2$, $g_y(1,2) = -2 - 4 = -6$, $\boxed{\operatorname{grad} g(1,2) = -2\vec{\imath} - 6\vec{\jmath}}$

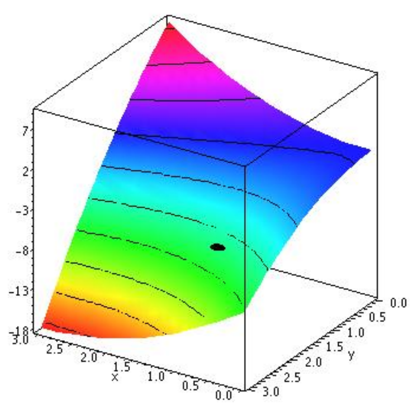
3. Compute $g_{\vec{v}}(1,2)$. Note: $\|\vec{v}\| \neq 1$.

$\vec{u} = \dfrac{1}{\sqrt{2}}(\vec{\imath} + \vec{\jmath})$,

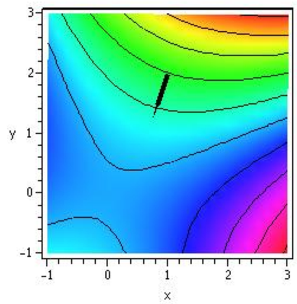
$\operatorname{grad} g(1,2) \cdot \vec{u} = (-2\vec{\imath} - 6\vec{\jmath}) \cdot (\frac{1}{\sqrt{2}}(\vec{\imath} + \vec{\jmath})) = \frac{1}{\sqrt{2}}(-2-6) = \boxed{-\dfrac{8}{\sqrt{2}} = -4\sqrt{2}}$

Problems 4 through 6 require you to print your Maple graphs. Please avoid printing more pages than necessary. Also, include some identification before printing so you can distinguish your printouts from the others.

4. Use Maple to graph the function $g(x,y)$. Mark the point on the graph where $x = 1$, $y = 2$ on a printout of this graph.



5. Now change your graph to contour style, and change the perspective to get a contour diagram of $g(x,y)$. On a printout of this graph, starting at the point (1,1), draw an arrow in the direction of the gradient vector $\operatorname{grad} g(1,1)$.



6. Let $f(x,y) = x^3y^3$

a. For each of the following points (a,b), compute grad $f(a,b)$: $(2,2), (2,-2), (-2,2), (-2,-2)$.

grad $f(x,y) = 3x^2y^3\vec{\imath} + 3x^3y^2\vec{\jmath}$

$\boxed{\text{grad } f(2,2) = 96(\vec{\imath} + \vec{\jmath})}$,

$\boxed{\text{grad } f(-2,2) = 96(\vec{\imath} - \vec{\jmath})}$,

$\boxed{\text{grad } f(2,-2) = 96(-\vec{\imath} + \vec{\jmath})}$,

$\boxed{\text{grad } f(-2,-2) = 96(-\vec{\imath} - \vec{\jmath})}$

b. Graph the function $f(x,y)$ in contour style and change the perspective to get a contour diagram of $f(x,y)$. For each of the points (a,b) listed above, mark the direction of the gradient vector you computed in part **a** on a printout of the graph of $f(x,y)$ starting at the point (a,b).

c. For each of the following $\vec{v}$, compute $f_{\vec{v}}(2,2)$

(i) $\vec{v} = \vec{i} + \vec{j}$

(ii) $\vec{v} = \vec{i} - \vec{j}$

Why stop there? These ideas generalize to functions of three variables: For function of three variables, $f(x,y,z)$:

$$\text{grad } f(a,b,c) = f_x(a,b,c)\vec{\imath} + f_y(a,b,c)\vec{\jmath} + f_z(a,b,c)\vec{k},$$

and the directional derivative of $f(x,y,z)$ at (a,b,c) in the direction of the unit vector $\vec{u} = u_1\vec{\imath} + u_2\vec{\jmath} + u_3\vec{k}$ is

$$f_{\vec{u}}(a,b,c) = \text{grad } f(a,b,c) \cdot \vec{u} = f_x(a,b,c)u_1 + f_y(a,b,c)u_2 + f_z(a,b,c)u_3$$

7. Let $f(x,y,z) = 3x^2y^3 + 4z^2$ Compute each of the following:

a. grad $f(1,2,-1)$

b. $f_{\vec{u}}(1, 2, -1)$, where $\vec{u} = 2\vec{\imath} + \vec{\jmath} - \vec{k}$.