Math 303, Worksheet #7 **Name:** ______________________
Fall 2008

A function $f(x, y)$, of two variables has two partial derivatives, f_x, and f_y. The derivatives of the partial derivatives are called **second-order partial derivatives** of $f(x, y)$. There are four of them:

$$f_{xx} = \frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) \qquad f_{xy} = \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)$$

$$f_{yy} = \frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) \qquad f_{yx} = \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)$$

Note that to compute f_{xy}, you first take the derivative with respect to x, then take the derivative with respect to y. The second order derivatives f_{xy} and f_{yx} are called **mixed partials**, and we saw that $f_{xy} = f_{yx}$.

---

1. Compute each of the four partial derivatives of $f(x, y) = 3x^2y - 4xy^3$, and check that, in fact $f_{xy} = f_{yx}$.

$f_{xx} =$

$f_{yy} =$

$f_{xy} =$

$f_{yx} =$

2. Compute each of the four partial derivatives of $g(x, y) = x^2 - x\ln(y^2) - 4y^3$, and check that, in fact $g_{xy} = g_{yx}$.

$g_{xx} =$

$g_{yy} =$

$g_{xy} =$

$g_{yx} =$

3. Compute the mixed partials for each of the following functions:

a. $f(x, y) = x^4 y^6$

$f_{xy} =$
$f_{yx} =$

b. $h(x, y) = x^m y^n$

$h_{xy} =$
$h_{yx} =$

c. $F(x, y) = G(x)H(y)$, where G is an arbitrary function of x, and H is an arbitrary function of y.

$F_{xy} =$
$F_{yx} =$

4. Let $p(x, y) = ax^2 + bxy + cy^2 + dx + ey + f$, where a, b, c, d, e and f are constants.

a. Compute:
$p_{xx} =$
$p_{yy} =$
$p_{xy} =$

b. Compute $D = p_{xx}p_{yy} - p_{xy}^2$.

It turns out that D determines the general shape of the graph of $p(x, y)$. Note that D depends only on the "quadratic part" of p, $ax^2 + bxy + cy^2$.

The general shapes for the graph of a quadratic function $p(x, y) = ax^2 + bxy + cy^2$ are: elliptical paraboloids like $z = x^2 + y^2$, hyperbolic paraboloids like $z = x^2 - y^2$, and parabolic cylinders like $z = x^2$. All others look like one of these graphs, except maybe stretched, shifted, rotated or flipped.

5. For each of the following functions, compute D using the formula you found in the previous problem. Then use Maple to graph the function, and determine its general shape:

(E) Elliptic Paraboloid;
(H) Hyperbolic Paraboloid; or
(C) Parabolic Cylinder.

a. $f(x, y) = 4x^2 + 2xy + y^2$

b. $q(x, y) = 4x^2 + 2xy - y^2$

c. $r(x, y) = -4x^2 + 2xy - y^2$

d. $s(x, y) = x^2 + 2xy + y^2$

e. $t(x, y) = x^2 + 4xy + y^2$

f. $u(x, y) = 2x^2 - 2xy + 3y^2$

6. There is a rule for determining the shape of a quadratic polynomial based on D. Can you guess what is it?