

For each of the following:

(i) Locate the critical points - places where the gradient is 0;

(ii) Compute the discriminant, $D = f_{xx}f_{yy} - f_{xy}^2$, then compute the value of D at each critical point.

(iii) Graph the function, and decide if each critical point is a local maximum, local minimum, a saddle point, or none of these.

1. $p(x, y) = x(y - 1)(x + 2)$

2. $q(x, y) = (x^2 - 1)(y^2 - 4)$

3. $f(x, y) = \frac{1}{x^2 + y^2 + 1}$

4. $g(x, y) = \frac{x}{x^2 + y^2 + 1}$

5. Each of the following functions is limited to the region $-2 \leq x \leq 2$ and $-2 \leq y \leq 2$. Use the graphs to determine

(iv) The x and y values where the function has maximum and minimum values, and

(v) The maximum and minimum value for the function.

a. $s(x, y) = x^2 + y^2$

b. $t(x, y) = x^2 + 3y^3$

c. $u(x, y) = x^2y - 3xy^3$

6. Your company makes two types of widgets. The demand functions are given by:

$$\begin{aligned}q_1 &= 500 - .2p_1 \\q_2 &= 400 + .1p_1 - .1p_2,\end{aligned}\tag{1}$$

where p_1 and p_2 are the prices of types 1 and 2, respectively, and q_1 and q_2 are the quantity of types 1 and 2 sold, respectively.

a. Why is the coefficient of p_1 negative in the formula for q_1 ?

b. Why is the coefficient of p_1 positive in the formula for q_2 ?

c. Find the formula for your total revenue (the amount of money you take in) as a function of p_1 and p_2 .

d. Find the critical point(s) for the revenue function.

e. Use the graph of the revenue function to determine whether each critical point is a maximum or a minimum.