Math 303, Worksheet #9 **Name:** _____
Fall 2008

Suppose $f(x, y)$ is a function and R is the rectangle determined by $a \leq x \leq b$ and $c \leq y \leq d$. Then

$$\int_R f dA = \int_{y=c}^{y=d} \int_{x=a}^{x=b} f(x, y)\, dx\, dy$$

This *iterated* integral can be computed by first computing $\int_a^b f(x, y)\, dx$, and then plugging the result into the integral with respect to y. Note that when we are integrating with respect to x, we treat y as a constant.

1. Compute $\int_R x^2 y\, dA$, where R is the rectangle $-2 \leq x \leq 2$, $0 \leq y \leq 4$.

You can compute $\int_{y=c}^{y=d} \int_{x=a}^{x=b} f(x, y)\, dx\, dy$ in Maple using the command:
$int(int(f(x, y), x = a..b), y = c..d)$

Sometimes you will need to use *evalf* to get Maple to give a numeric answers.

2. Compute each of the following:

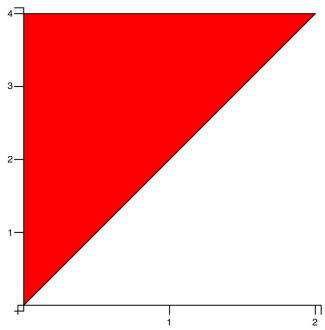
a. $\int_R \sin(x^2 + y)\, dA$, where R is the rectangle $0 \leq x \leq 2$, $0 \leq y \leq \pi$.

b. $\int_R \sqrt{x^2 + y^2}\, dA$, where R is the rectangle $-2 \leq x \leq 2$, $0 \leq y \leq 4$.

3. Compute $\int_R x \sin y\, dA$, where R is the rectangle $0 \leq x \leq 2$, $-\pi \leq y \leq \pi$. Use the graph of $f(x, y) = x \sin y$ to explain your answer.

In order to compute an integral over a region that is not a rectangle, we will need to be able to describe the region using inequalities. For example, the region below can be described by:

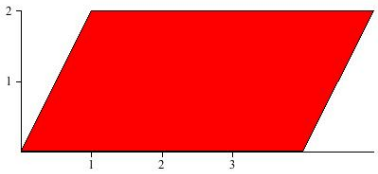
$0 \leq x \leq 2$ and $2x \leq y \leq 4$. (I used the fact that the diagonal line of the triangle is the line $y = 2x$, the top line is the line $y = 4$, and the left line is the line $x = 0$).



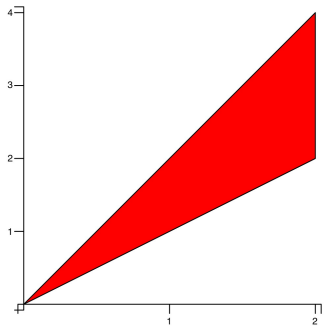
Note that by rewriting $y = 2x$ as $x = \frac{1}{2}y$ we could have described this triangle by $0 \leq y \leq 4$ and $0 \leq x \leq \frac{1}{2}y$.

4. Describe the following regions using inequalities. You might want to start by finding equations for each of the sides of the region.

a.



b.



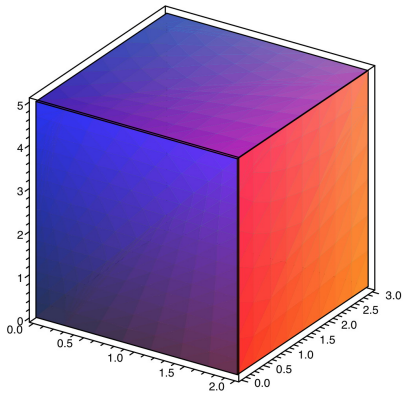
5. Sketch the region determined by the inequalities $1 \leq x \leq 4$ and $1 \leq y \leq x$.

6. Sketch the region determined by the inequalities $0 \leq x \leq 4$ and $0 \leq y \leq x^2$.

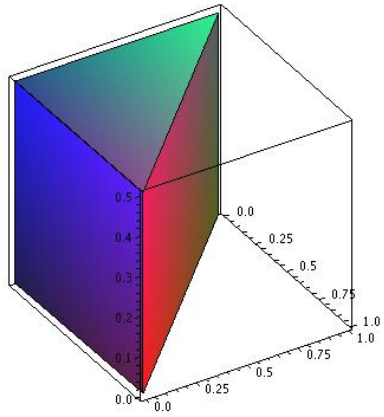
7. Sketch the region determined by the inequalities $1 \leq x \leq 4$ and $x \leq y \leq x^2$.

8. We can also describe 3d objects using inequalities. Describe the following regions using inequalities. You might want to start by finding equations for each of the sides.

a.



b.



c.